Heuristic Algorithms for Solving Convex and Nonconvex Economic Dispatch

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***Abstract*—Economic dispatch (ED) is a power system optimization problem and its objective is to reduce the total generation cost of units while satisfying constraints. The presence of nonlinearities in practical generator operation makes solving the ED problem more challenging. These generator nonlinearities are modeled as constraints to be met in the form of ramp-rate limits and prohibited operating zones. This paper proposes three heuristic algorithms, namely, the genetic algorithm (GA), differential evolution (DE) and modified particle swarm optimization (MPSO) to solve this ED problem for two test systems. Simulation, numerical results and convergence performances of these three algorithms are presented and compared as a way of demonstrating and validating the heuristic algorithms in solving this complex and challenging power system problem characterized by practical and nonconvex generator constraints.**

Keywords-Differential evolution; economic cost function; economic dispatch; generation cost; genetic algorithm; particle swarm optimization; prohibited operating zones; ramp-rate limits.

NOMENCLATURE

a_i, b_i & c_i	Fuel cost coefficients for unit i
β_{gb}	Global best strategic learning parameter
c_1 & c_2	Cognitive and social acceleration constants respectively
d	Particle's dimension
C_R	Crossover constant
S_i	Generator status indicator
F	Scaling factor for mutation
fit	Fitness
i	Index of running generating units
$Iter$ & $Iter_{max}$	Current and maximum iteration numbers respectively
t	Continuous time step
K_{pb}	Penalty factor coefficient for ED real power balance constraint
l	lth particle
np	Number of population in a generation
N	Total number of online generating units
P_{gd}	Swarm's best position for dimension d
P_{lbd}	lth particle best position for dimension d
P_{ld}	Position vector of the particle l in dimension d
P_i	Generating capacity of unit i
P_{loss}	System loss
P^D	Total real load demand
r, $rand$, $rand_1$ & $rand_2$	Random numbers with uniform distribution in the range of [0, 1]
$randn$	Gaussian distributed random number with a zero mean and a variance of 1
U^O	Initial random population
U_1, U_2 and U_3	Randomly selected parents population vectors
U_3', U_i & U_t	Offspring, primary array and trial vectors respectively
V_{ld}	lth particle velocity in dimension d
w, w_{max} & w_{min}	Current, final and initial inertia weights respectively

This work was supported by the National Science Foundation, USA under Grant ECCS # 0348221.

I. INTRODUCTION

Economic dispatch (ED) is one of the important optimization problems in power systems that have the objective of dividing the power demand among the online generators economically while satisfying various constraints [1]. Modern power system is experiencing increased demand for electricity with related expansions in system size, which has resulted in higher number of generators and lower reserve margins making the ED problem more challenging and complicated [1]. Conventional dispatch algorithms employ Lagrangian multipliers and require monotonically increasing cost curves. Unfortunately, the input-output characteristics of modern generating units are inherently highly nonlinear due to valve-point loading effects, ramp-rate limits, prohibited operating zones and so on, which tend to generate multiple local minima points in the cost function [2], [3]. Classical dispatch algorithms require that these characteristics be approximated, however such approximations may lead to suboptimal operation of the generator and results in heavy revenue losses.

The basic ED considers the power balance constraint apart from the generating capacity limits. However, a practical ED must take ramp-rate limits, prohibited operating zones, valve-point effects, and multi-fuel options into consideration to provide the completeness for the ED formulation. The resulting ED is a nonconvex optimization problem, which is a challenging one and cannot be solved by traditional methods. Some factors that influence ED of the system are operating efficiency of generating units, fuel and operating costs, and transmission losses [1], [2]. The ED problems are in general nonconvex optimization problems with many local minima. Numerous classical techniques such as Lagrange based algorithms, linear programming (LP), non-linear programming (NLP) and quadratic programming (QP)and dynamic programming (DP) algorithms have been reported in the literature [4], [5].

Optimization problems in power system including ED have complex and nonlinear characteristics with stringent equality and inequality constraints to be satisfied. Different optimization techniques applied so far to solving these problems can be classified according to the type of the search space and/or the objective function [4] - [6]. Depending on the problem formulation, the objective function could be minimization of the unit generation costs, maintenance costs or some predefined reliability risks subject to some constraints resulting in nonlinear optimization as proposed in [4] - [6]. Solving such nonlinear optimization problems for most cases may not be feasible because their numerical solutions require extensive computational efforts, which increase exponentially with the problem complexities. Even though deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters [6], [7].

This paper presents three heuristic algorithms, namely, genetic algorithm (GA), differential evolution (DE) and modified particle swarm optimization (MPSO), used to solve these highly challenging ED problems of two test systems whose generating units are characterized by convex and nonconvex operational features. Solving this practical optimization problem leads to a minimized total generation cost of operating the two respective power systems in the presence of generator constraints.

The primary contributions of this paper are:

- Application of GA, DE and MPSO algorithms to solving the ED problem of two test systems with convex and nonconvex fuel cost functions.
- Comparison of performances of GA, DE and MPSO based ED algorithms for minimization of economic costs objective function.
- Application of the heuristic algorithms to solving the nonconvex ED problems where the classical Lagrange based algorithm cannot be directly applied.
- Solving GA, DE and MPSO based ED problems considering other practical generator constraints, namely, load balance, generator ramp-rate limits, prohibited operating zone and spinning reserve.

II. PROBLEM FORMULATION OF ED

The ED problem is to find the optimal combination of power generations that minimizes the total generation cost while satisfying some constraints. The ED problem is commonly formulated as costs optimization problems, with the aim of minimizing the total generation cost of the power system but still satisfying equality and inequality constraints.

The objective here is to minimize the economic cost function expressed as second order function of each unit output P_i as shown in (2) subject to satisfying practical generator constraints.

The prohibited operating zones in the input-output performance curve for a typical thermal unit can be due to vibrations of shaft bearing caused by a steam valve or the associated auxiliary equipment such as boilers, feed pumps etc [1], [8]. Practically, the shape of input-output curve in the neighborhood of a prohibited zone is difficult to determine by actual performance testing or from operational records. Hence in actual operation, the best economy is achieved by avoiding the operation in these areas [1], [8]. Cost function or the constraints can be modeled to include the prohibited operating zones according to the description shown in Fig. 1.

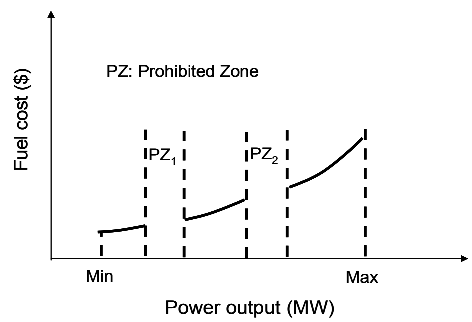

Fig. 1. Example of cost function with two prohibited operating zones

It is commonly assumed that the generation output can be adjusted instantaneously. Although this assumption is useful because it simplifies the problem, it however does not reflect the actual operating process of the generating unit. The operating range for online generating units is practically restricted by their ramp-rate limits [1], [2], [8].

The inclusion of the prohibited zones, ramp-rate limits and other practical constraints results in nonconvex ED of generating units. The practical ED considering generator nonlinearities outlined above are modeled in this paper.

Define

$$S_i = \begin{cases} 1 & \text{if unit } i \text{ is offline} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The objective function can be expressed as

$$\min F = \sum_{i=1}^{N} \left\{ \left(a_i + b_i P_i + c_i P_i^2 \right) \left(1 - S_i \right) \right\} \quad (2)$$

subject to the following ED constraints:

- *Load balance*

The generated power from all the running units must satisfy the load demand and the system losses given by (3).

$$\sum_{i=1}^{N} P_i = P^D + P_{loss} \tag{3}$$

To calculate system losses, algorithms based on penalty factors and constant loss formula coefficients or the B-coefficients are in use [1], [8]. System losses expression based on the B-coefficients is used in this paper and is given by (4).

$$P_{loss} = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{oi} P_i + B_{oo} \tag{4}$$

- *Generator ramp-rate limits*

The power output of a practical generator cannot be adjusted instantaneously without limits. The operating range of all online units is restricted by their ramp-rate limits during each dispatch period. Therefore, the subsequent dispatch output of a generator should be limited between the constraints of up and down ramp-rates [1], [2], [8], hence the generator operating limits given by (5) are modified according to (6).

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5}$$

$$\max(P_i^{\min}, P_i^{pre} - DR_i) \le P_i \le \min(P_i^{\max}, P_i^{pre} + UR_i) \tag{6}$$

- *Prohibited operating zone*

Each generator has its generation capacity, which cannot be exceeded at any time. It is common for a typical thermal unit to have a steam valve in operation, or a vibration in a shaft bearing, which may result in interference and discontinuous input-output performance-curve sections [1], known as the prohibited zones, as shown in Fig. 1. Practically, adjusting the power output of a unit must avoid all capacity limits and unit operations in prohibited zones [8]. The acceptable operating zones of a generating unit can be formulated as shown in (7).

$$\left.\begin{aligned} &P_i^{\min} \le P_i \le P_{i,1}^{lower} \\ &P_{i,j-1}^{upper} \le P_i \le P_{i,j}^{lower} \qquad j = 2,3,\ldots, pz_i \\ &P_{i,pz_i}^{upper} \le P_i \le P_i^{\max} \end{aligned}\right\} \tag{7}$$

- *Spinning reserve constraint*

Sufficient spinning reserve is required from all running units to maximize and maintain system reliability.

$$\sum_{i=1}^{N} P_i^{\max} \ge P^D + R \tag{8}$$

III. Economic Dispatch Problem

The load demand is distributed among the running units in ED. The generation output of each unit should lie between the minimum and maximum power limits for good ED [1]. While minimizing the total generation cost, the total generation from running units should be equal to the total system demand plus the transmission network loss. The ED consists of finding the optimum operating policy and distribution of power among the running units while satisfying constraints (3 - 7) [1].

The evaluation function *f* (which is also called the fitness function in evolutionary algorithms), is defined for evaluating the fitness of each individual (or particle) in the generation (or swarm). The penalty function algorithm uses functions to penalize the objective function or the fitness of the individual (or particle) in proportion to the magnitude of the constraint violation [1]. The penalty function parameter is selected to distinguish between infeasible and feasible solutions.

In order to emphasize the 'best' solution and speed up convergence of the iterative procedure, the evaluation function *f* is defined according to (9) to minimize the fuel cost function given by (2) for a specified load demand P^D while satisfying the system constraints.

$$f = F + K_{pb}\left(\sum_{i=1}^{N} P_i - P^D - P_{loss}\right)^2 \tag{9}$$

In order to limit the evaluation value of a potential solution within a feasible range, the generators' real output power operating limits constraint in (7) should be satisfied. If a potential solution satisfies this constraint, then it is a feasible solution and *f* has a relatively minimal evaluation value *f*. Otherwise, the *f* value of this potential solution is penalized.

A. General Pseudo Code for GA, DE and MPSO based ED Algorithms

*Step*1: Specify the generation power limits, ramp-rate limits and prohibited operating zones of each unit.
Step 2: Set all parameters
Step 3*:* Select GA, DE or MPSO?
Step 4: Initialize randomly the chromosomes/ individuals/ particles in a generation/ iteration to lie within the specifications in *Step* 1.
Step 5: Evaluate the fitness values of all chromosomes/ individuals/ particles.
Step 6: Perform selection, crossover and mutation to determine best fit chromosome/ individual in a generation for GA or DE, or determine *pbest* and *gbest* of the particles' population, and perform mutation on *gbest* for MPSO.
Step 7: Update the velocity and position of each particle in MPSO.
Step 8: Determine the best chromosome/individual/global best particle.
Step 9: Terminate and print results if maximum generation/ iteration is reached. Otherwise, go to *Step* 4.

B. GA Based Economic Dispatch

Genetic algorithm is a search algorithm based on the modeling of natural genetics and natural selection [5] – [7]. The three main operators in used in GA are reproduction, crossover and mutation.

Implementation of a problem in a GA starts from the parameter encoding (that is, the representation of the problem). The encoding must be carefully designed to utilize the GA's ability to efficiently transfer information between chromosome strings and objective function of the problem. The GA for this ED problem is encoded by grouping the on-line units for ED according to Fig. 2. Each chromosome consists of maximum number of units as genes, with each gene encoded as 16 bits. As shown in Fig. 2, each generating unit power output is concatenated and encoded in a binary based string normalized over its operating range. In this paper, each generating unit string is assigned by 16 bits, thus a string individual has $16 \times N$ bits [7].

To obtain the actual power output of each generating unit for fitness evaluation, each string is decoded to the decimal value using (9) [7].

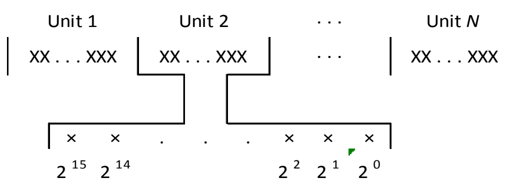

Fig. 2 $16 \times N$ bits concatenated binary coding scheme of generating units

Evaluation of a chromosome is achieved by decoding the encoded chromosome string using (10) and then computing its fitness.

$$P_i = P_i^{\min} + \frac{D_i \times \left(P_i^{\max} - P_i^{\min}\right)}{2^{16} - 1}, \quad \text{for } i=1, 2, \ldots, N \tag{10}$$

Where (10) ensures that the generating units' output power P_i lies within the units' operating real power limits.

C. DE Based Economic Dispatch

Differential evolution is an optimization algorithm that solves real-valued problems based on the principles of natural evolution [5]. Like other evolutionary algorithms, DE also relies on initial random population generation, which is then improved using selection, mutation, and crossover repeated through generations until the convergence criterion is met [5].

Although the canonical form of differential evolution solves optimization problems over continuous spaces, minor adjustments to the code allow DE to solve mixed integer optimization problems [5]. This is achieved with the use of operator that rounds the variable to the nearest integer value, when the value lies between two integers.

An initial population composed of vectors U_i^o, $i = 1,2,\ldots np$, is randomly generated within the parameter space. In each generation, *np* competitions are held to determine the composition of the next generation. Every pair of randomly chosen vectors U_1 and U_2 defines a vector differential: $(U_1 - U_2)$. Their weighted differential is used to perturb another randomly chosen vector U_3 according to (11) given by:

$$U_3' = U_3 + F * (U_1 - U_2) \tag{11}$$

Typically $(0 \leq F \leq 1.0)$ and it controls the speed and robustness of the search; a lower value increases the rate of convergence but also the risk of being stuck at the local optimum. The crossover is a complimentary process for DE. It aims at reinforcing the prior successes by generating the offspring vectors. In every generation, each primary array vector U_i, is targeted for crossover with a vector like U_3' to produce a trial vector U_t according to (12).

$$U_t = \begin{cases} U_3' & \text{if } rand < C_R \\ U_i & \text{otherwise} \end{cases} \tag{12}$$

The newly created vector is evaluated by the objective function and the corresponding value is compared with the target vector. The best fit vector is kept for the next generation as given by (13). The best parameter vector is evaluated for every generation in order to track the progress made throughout the minimization process; thus making the DE elitist algorithm.

$$U_i(t+1) = \begin{cases} U_i(t) & \text{if } fit(U_i(t)) \leq fit(offspring\ (t)) \\ offspring & \text{otherwise} \end{cases} \tag{13}$$

The best parameter vector evaluated for the latest generation produce the economic dispatch generation with the minimum generation cost [8], [9].

D. MPSO Based Economic Dispatch

The modified PSO is a combination of PSO and an evolutionary strategy enhancing the method to perform optimal search under complex environments [10]

Let X and V denote a particle coordinates (position) and its corresponding flight speed (velocity) in a search space, respectively. Therefore, the lth particle is represented as $X_{ld} = (X_{l1}, X_{l2},\ldots, X_{lN})$ in the d-dimensional space. The best previous position of the lth particle is recorded and represented as $P_{lbd} =(P_{lb1}, P_{lb2},\ldots, P_{lbN})$. The index of the best particle among all the particles in the group is represented by the P_{gd}. The rate of the velocity for particle lth is represented as $V_{ld}=(V_{l1}, V_{l2},\ldots,V_{lN})$. In this version of PSO, the velocity is limited to a certain range $[-V_{max}, V_{max}]$ such that V_{ld} always lies in that range [10]. The new velocity and position for each particle i in dimension d is determined according to the velocity and position update equations given by (14) and (15), while the inertia weight is updated according to (16).

$$\begin{aligned} V_{ld}(t) &= w \cdot V_{ld}(t-1) + c_1 \cdot rand_1 \cdot (P_{lbd}(t-1) - X_{ld}(t-1)) \\ &+ c_2 \cdot rand_2 \cdot (P_{gb}^{*}(t-1) - X_{ld}(t-1)) \end{aligned} \tag{14}$$

$$X_{ld}(t) = X_{ld}(t-1) + V_{ld}(t) \tag{15}$$

$$w = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{iter_{\max}}\right) \times iter \quad (16)$$

The MPSO is a combination of PSO and an evolutionary strategy enhancing the algorithm to perform optimal search under complex environments [10]. This version of MPSO is a variant of the original formulation of the continuous particle swarm optimization (CPSO) to solve continuous optimization problems such as the ED problem considered in this paper. Supposing P_{gd} is the swarm's global best particle chosen with a random number less than a predefined mutation rate (for 0 < mutation rate < 0.2) then the mutation result of this particle is given by (17).

$$P^{*}_{gd}(t-1) = P_{gd}(t-1) + ceil(randn \times P_{gd}(t-1) / \beta_{gb}) \quad (17)$$

d=1, 2, …, N

where β_{gb} is global best strategic learning parameter for mutation that can be either dynamically changing or fixed and controls the mutation process introduced in this MPSO algorithm. The main goal is to increase the diversity of the population by preventing the particles from moving too close to each other, thus converging prematurely to local optima. This in eventually improves the CPSO's search performance.

The control or the decision variables for the ED problems are real power generations, and are therefore used to form the swarm. The real power outputs of all on-line generators are represented as the positions of the particles in the swarm [11] – [14]. For N on-line generators, the particle position is represented as a vector of length N. If there are N_{par} particles in the swarm, the lth particle in the swarm can be represented as a matrix shown in (18).

$$P_{ld} = [P_{l1}, P_{l2}, ..., P_{li}, ..., P_{lN}\ ] \quad (18)$$

d=1, 2, …, i, …, N

Where P_{ld} is the position vector of the particle l in dimension d and represents a potential solution to the optimization problem. The element P_{li} of the P_{ld} vector is the lth position component of particle l, and it represents the real power generation of on-line generator i of the possible solution.

Each element of the swarm matrix is initialized randomly within the effective real power operating limits. The initialization is either based on (5) for generators without ramp-rate limits, or on (6) for generators with ramp-rate limits which is the case considered in this paper.

The ith dimension of the lth particle is assigned a value of P_{li} given by (19) to satisfy the constraint given by (6).

$$P_{li} = P_{li}^{\min} + r * (P_{li}^{\max} - P_{li}^{\min}) \quad (19)$$

The fitness values obtained from (9) for the initial particles of the swarm are set as the initial *pbest* values of the particles. The best value among all the *pbest* values becomes the *gbest*. Mutation operator is introduced into the algorithm using (17).

The new velocity is computed using (14). To control excessive roaming of particles, the velocity is limited to a certain range [- V_{max}, V_{max}] such that V_{ld} always lies in that range. The maximum velocity is limited to between 10% - 20% of the dynamic range of the variable on each dimension.

The swarm is updated by updating the particle's position vector using (15). The *pbest* and *gbest* values are subsequently updated.

IV. Case Studies, Numerical results and Analysis

A. Case Studies

Two case studies are presented in this section of the paper for solving the convex and nonconvex static economic dispatch problems using heuristic algorithms.

Case I

The test system for this case consists of 6 thermal units, including the generation limits, fuel cost coefficients, ramp-rates limits and prohibited operating zones of these thermal units, 26 buses and 46 transmission lines [14] - [16]. The system real power loss is also considered in this case. The loss coefficients on 100 MVA base capacity are presented in (20 - 22). The total load demand is 1263. There is 207MW of total spinning reserve accruable from the 6 thermal units, amounting to 14.08% of total generation thus satisfying the constraint in (8).

Using the 6-unit data, the total generation cost resulting from the online units can be evaluated based on their economic dispatch generation after minimization of the objective function in (2) subject to the constraints (3 - 8).

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{bmatrix} \quad (20)$$

$$B_{oi} = 1.0e^{-03} * [-0.3908 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad 0.2161 \quad -0.6635] \quad (21)$$

$$B_{oo} = 0.0056 \quad (22)$$

Case II

The test data for this case is taken from a real power system consisting of 19 generating units characterized by convex fuel cost coefficients, taken from two industrial parks located in Bintan and Batam in Indonesia [17]. The total generation cost is evaluated as described in Case I.

B. Numerical Results and Analysis

All numerical results are obtained based on programs developed using Matlab environment and ran on PC with

2.2GHz CPU speed and 1.5GB of RAM. Results of the two cases are presented below.

The following parameters are used by the three heuristic algorithms:

GA

Generation size of 50 chromosomes (or individuals), maximum generation number of 500 and 300 for Cases I and II respectively, crossover rate of 0.7, linearly decreasing mutation rate (from 0.7 to 0.1), β_{gb} of 2 and K_{pb} is empirically tuned to value of 1000 for the two cases.

DE

Generation size of 50 individuals, maximum generation number of 500 and 300 for Cases I and II respectively, scaling factor for mutation is 0.5, crossover rate of 0.8, β_{gb} of 2 and K_{pb} is empirically tuned to value of 1000 for the two cases.

MPSO

Population size of 30, w_{min} and w_{max} of 0.4 and 0.9 respectively, c_1 and c_2 of 2 each, V_{max} is 20% of the dynamic range of the variable on each dimension, mutation rate of 0.15, $Iter_{max}$ of 500 and 300 for Cases I and II respectively, β_{gb} of 2 and K_{pb} is empirically tuned to value of 1000 for the two cases.

Case I

Table I shows the ED schedules generated by each of the 6 thermal units, total generation, total power loss and total generation costs produced by GA, DE and MPSO based ED algorithms. The table shows the amount of power generation economically dispatched to meet the load demand of 1263 MW while satisfying constraints (3 - 8). Total minimum generation costs in meeting load demand, as produced by GA, DE and MPSO based ED algorithms are $15450, $15446 and $15444 respectively, while the real power losses produced are 12.3434MW, 12.4900MW and 12.0141MW respectively. Dispatch results from MPSO based ED algorithm is seen to result in the least real power loss compared to the GA and DE based ED algorithms. The MPSO algorithm also generated the most economical and minimal total generation cost compared with the other two algorithms. These results demonstrate MPSO based ED algorithm's better ability in solving the ED problem compared with the GA and DE based ED algorithms for units characterized by ramp-rate limits and prohibited operating zones. The GA algorithm on the other hand generated lower real power loss compared with the DE algorithm.

Table II shows the statistical comparison of the generation costs among all the ED algorithms considered in this paper. The table shows MPSO performing better than the GA and DE based ED algorithms in terms of the magnitude of variations in minimum generation costs and standard deviations. The results presented in Tables I and II shows a reduction in both the total generation costs and transmission losses produced by GA, DE and MPSO in this paper compared with the results presented in [14] – [16] on the same test system. The significant power loss reductions with the MPSO presented in this paper have the benefits of energy saving, fuel cost reduction, and also emission of generated-unused power is reduced. The loss reductions are reflections of improvements in the performances of the algorithms presented in this paper.

TABLE I
POWER GENERATION AND GENERATION COSTS DATA
FOR THE 6-UNIT TEST SYSTEM

Generating units	Algorithms							
	100 Trials (Proposed)			50 Trials [14]		100 Trials [15]	50 Trials [16]	
	GA	DE	MPSO	GA [14]	PSO [14]	NPSO-LRS [15]	PSO [16]	MPSO [16]
P_1	439.7774	445.5862	438.9866	474.8066	447.4970	446.9600	451.2741	444.8882
P_2	179.6234	187.4714	162.9011	178.6363	173.3221	173.3944	162.4633	168.1455
P_3	261.5945	261.7543	267.0032	262.2089	263.4745	262.3436	262.6419	265.0000
P_4	133.5927	130.3120	138.7787	134.2826	139.0594	139.5120	130.3146	129.4751
P_5	151.3102	160.1349	158.3033	151.9039	165.4761	164.7089	173.8361	173.0299
P_6	109.4452	90.2312	109.0412	74.1812	87.1280	89.0162	95.1188	95.0435
Total generation over an hour (MW)	1275.3434	1275.4900	1275.0141	1276.0300	1276.0100	1275.9351	1275.6488	1275.5823
Load supplied over an hour (MW)	1263.0000	1263.0000	1263.0000	1263.0000	1263.0000	1263.0000	1263.0000	1263.0000
P_{loss} over an hour (MW)	12.3434	12.4900	12.0141	13.0217	12.9584	12.9361	12.6448	12.6411
Total min. generation cost over an hour ($)	15450	15446	15444	15459	15450	15450	15446	15444

TABLE II
STATISTICAL COMPARISON OF GENERATION COSTS
FOR THE 6-UNIT TEST SYSTEM

Algorithms		Total generation cost over an hour			
		Min. ($)	Max. ($)	Ave. ($)	Std
100 Trials (Proposed)	GA	15450	15498	15478	25.2104
	DE	15446	15501	15483	28.3469
	MPSO	15444	15496	15457	24.8425
50 Trials [14]	GA [14]	15459	15524	15469	-
	PSO [14]	15450	15492	15454	-
100 Trials [15]	NPSO-LRS [15]	15450	-	-	-
50 Trials [16]	PSO [16]	15446	15538	15477	-
	MPSO [16]	15444	15504	15460	-

Table III shows the statistical comparison of computation efficiency for all the three ED algorithms considered in this paper. The MDPSO based ED algorithm demonstrates faster computation time in locating minimum generation cost compared with GA and DE based ED algorithms as numerically shown in Table III for various numbers of maximum generations/iterations. The GA however, converged to global solutions at faster rate than the DE under similar operating conditions as seen in Table III.

TABLE III
STATISTICAL COMPARISON OF COMPUTATION EFFICIENCY
FOR THE 6-UNIT TEST SYSTEM

Description	Algorithms	Generations/Iterations				
		100	500	1000	3000	5000
Total CPU time (sec)	GA	2.51	11.58	23.88	84.85	175.32
	DE	9.19	44.71	92.11	284.96	510.68
	MPSO	0.59	1.63	3.05	8.55	14.29

Fig. 3 shows the minimum generation costs convergence performance for the GA, DE and MPSO based ED algorithms. The converged result conforms to the minimum generation costs presented in Tables I and II.

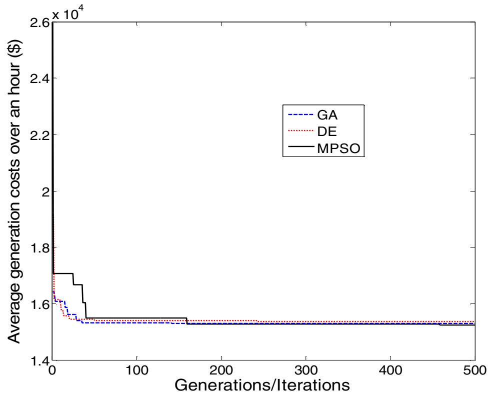

Fig. 3. Generation cost plots for the 6-unit test system

Case II

The ED schedules generated by each of the 19 online generating units using GA, DE and MPSO based ED algorithms are presented in Table IV. The table shows the units' generation economically dispatched to balance up the supplied load as shown in Table IV while meeting the system constraints. The result shows that the DE and MPSO performed comparably well in terms of finding the most economical dispatch generation, total generation and minimum generation costs. Percent deviation errors in perfectly matching the supplied load of 70.2 MW produced by GA, DE and MPSO based ED algorithms are 0.062%, 0.066% and 0.00% respectively.

TABLE IV
POWER GENERATION AND GENERATION COSTS DATA
FOR THE 19-UNIT TEST SYSTEM

Generating units	Generation (MW)		
	GA	DE	MPSO
P_1	5.8720	5.4398	4.1000
P_2	1.7407	5.8829	3.6000
P_3	5.8214	5.3230	6.1000
P_4	4.5662	1.1908	3.1000
P_5	4.9589	4.6090	3.1000
P_6	4.7991	4.9791	6.1000
P_7	2.8216	4.2533	6.1000
P_8	1.9510	4.8229	5.1000
P_9	3.8598	3.1180	2.1000
P_{10}	5.9470	3.5836	3.4000
P_{11}	2.6318	4.1133	2.4000
P_{12}	1.7762	3.9459	4.4000
P_{13}	4.8375	6.1180	5.1000
P_{14}	7.1800	5.4582	6.0000
P_{15}	2.5943	0.7223	2.1000
P_{16}	1.0058	1.5431	1.1000
P_{17}	1.7053	1.0481	2.1000
P_{18}	0.8738	1.8333	2.1000
P_{19}	5.3010	2.2615	2.1000
Total gen. over a week (MW)	70.2434	70.2461	70.2000
Load supplied over a week (MW)	70.2000	70.2000	70.2000
Total generation cost over a week ($)	242220	242240	242210

Table V shows the statistical comparison of the generation costs among all the ED algorithms considered. The table shows GA and MPSO algorithms performing fairly better than the DE algorithm in the statistical variation of the generation costs arrived at and its corresponding standard deviations.

Table VI shows the statistical comparison of computation efficiency for the GA, DE and MPSO based ED algorithms for the 19 units test system for different maximum generations/iterations. The data in table VI are shown plotted in Fig. 5.

TABLE V
STATISTICAL COMPARISON OF GENERATION COSTS
FOR THE 19-UNIT TEST SYSTEM

Algorithms	Total generation cost over a week			
	Min. ($)	Max. ($)	Ave. ($)	Std
GA	242220	242310	242300	28.4032
DE	242240	242340	242320	31.5620
MPSO	242210	242350	242280	40.0371

TABLE VI
STATISTICAL COMPARISON OF COMPUTATION EFFICIENCY
FOR THE 19-UNIT TEST SYSTEM

Description	Algorithms	Generations/Iterations				
		100	500	1000	3000	5000
Total CPU time (sec)	GA	34.94	87.63	136.52	509.10	999.32
	DE	63.73	199.24	330.16	815.24	1363.89
	MPSO	2.28	7.57	14.33	39.72	67.66

The convergence performance of the GA, DE and MPSO based ED algorithms for the 19 units test system is presented in Fig. 4. The figure shows the minimum generation costs convergence behavior for each of the three algorithms considered in this paper which simply corresponds to the best generation costs desired, and conforms to the result presented in Tables IV and V. The MPSO and the GA algorithms generated better minimum generation costs than the DE algorithm.

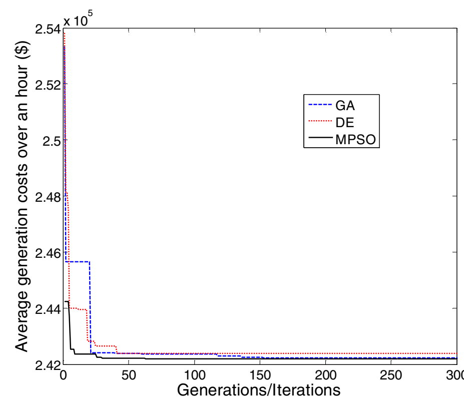

Fig. 4. Generation cost plots for the 19-unit test system

Fig. 5 presents plots for comparison of the computation efficiencies of the GA, DE and MPSO based ED algorithms for the 6 and 19 units test systems. For the two test systems, the MPSO shows faster computation efficiency compared with any of the other two algorithms. The DE presents slower computation efficiency compared with the GA algorithm, even as the problem dimension is increased.

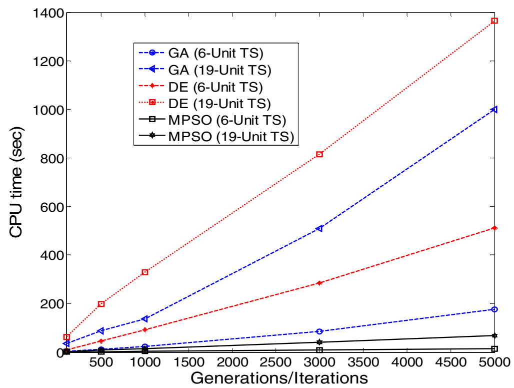

Fig. 5. CPU time versus generations/iterations

V. Conclusion

The economic dispatch (ED) problems for generators exhibiting practical and nonconvex characteristic behavior have been illustrated. Transmission line losses are incorporated to test the robustness of the heuristic algorithms. The heuristic algorithms have been compared and successfully applied in solving the ED problems of two practical power systems. The modified particle swarm optimization (MPSO) algorithm shows better performance in terms of the quality of results, loss reduction and computational efficiency in locating optimal solution when compared with the genetic algorithm (GA) and differential evolution (DE) under similar operating conditions. Increased problem dimensionality is seen not to have limitation on the algorithms' performances and in the quality of results obtained for the two power systems considered in this paper. The results offer good alternative for power system operational and planning activities in control areas desiring optimized energy management, generation costs curtailment and transmission loss reduction, in the face of continuously increasing global fuel costs and the irreplaceable depletion of conventional raw fuel resources.

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