A New Probabilistic Load Flow Method Using MCMC in Consideration of Nodal Load Correlation

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***Abstract*—This paper proposes a new method for the probabilistic load flow calculation. In this paper, a hybrid method of deterministic annealing expectation maximization (DAEM) algorithm, Markov Chain Monte Carlo (MCMC) and the AC load flow is presented to evaluate the effect of uncertainties of input variables** on the output ones**. DAEM is effective for estimating the maximum likelihood estimate (MLE) of probability density function (PDF) while maintaining the non-Gaussianity and the nonlinear correlation of the variables. DAEM is an extended algorithm of EM that calculates estimates for incomplete data. MCMC is used to generate the samples from arbitrary distribution while reflecting the non-Gaussianity and the nonlinear correlation of PDF. The proposed method is successfully applied to a sample system with real data.**

Keywords-Probabilistic Load Flow; Uncertainty; Deterministic Annealing Expectation Maximization; Maximum Likelihood Estimation; Markov Chain Monte Carlo; Non-linear Correlation; Multivariate Gaussian Mixture Distribution

I. INTRODUCTION

This paper presents a new probabilistic load flow (PLF) method using Markov Chain Monte Carlo (MCMC) with the preprocessing of deterministic annealing expectation maximization (DAEM) algorithm. The load flow computation has been widely applied to power system operation[1][2] and planning[3]-[5]. The conventional load flow is a deterministic technique in which the accuracy is dependent on a prior knowledge in handling a set of scenarios for power system conditions[6]. The uncertainty of the input variables such as generation, load and network conditions are not be reflected in the output results such as bus voltages or line flows since the input values are treated as the specified ones. However, in recent years, the difficulty in forecasting the generation and load is increasingly growing due to the liberation of power networks and the emergence of distributed generators[3][4][7][8]. Because the uncertainties affect the operation and planning, the analysis tools are required to handle the assessment of power systems under the uncertain conditions[9][10]. PLF approach is one of the techniques for assessing the uncertainty of power systems. It is possible to find the potential risks and the weak points of power systems through evaluating the uncertainties of the output variable like Fig. 1[7][11]-[13] where the relationship between the electrical load and relative frequency is depicted to allow operators to evaluate risks. Many PLF methods have been proposed to study the uncertainties of load flow. These methods may be classified as Monte Carlo Simulation Method (MCS-LF)[11], analytical method[2][7][8][12], hybrid method[13], *etc*.

MCS-LF is a direct technique for the PLF problem[12]. Theoretically, without the assumptions and constraints in MCS-LF, it is easy to evaluate the effect of the correlations of input variables and the uncertainty of network topology through the nonlinear equation. In MCS-LF, the input scenarios are generated from each probabilistic distribution of input variables. Next, the output scenarios are calculated with the deterministic load flow (DLF). Finally, the probabilistic characteristics of output variables are obtained through evaluating the output scenarios. Due to the use of the DC flow, the solution is sensitive to a prior knowledge of input variables. Therefore, it is expected that the obtained results become more accurate as appropriate input information is available. However, because the method requires a large computational effort and memory to obtain significant results, MCS-LF with simple random sampling (SRS)[16][17] was carried out to examine the accuracy[2][7][11]-[16]. Recently, according to the development of the computer hardware and MCMC sampling technique, it is possible to obtain more exact results with MCMC sampling than with SRS[10][11].

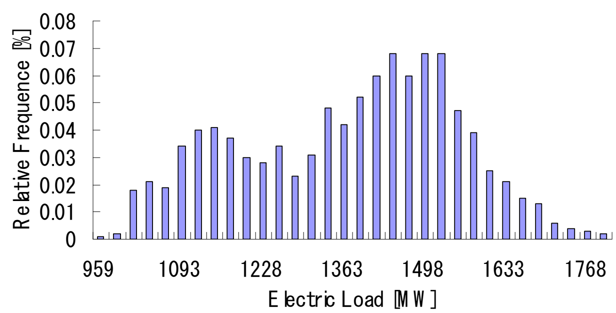

Figure 1. Frequency Distribution of Electric Hourly Load[20]

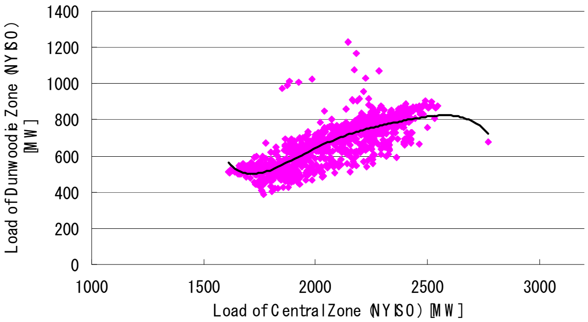

Figure 2. Correlation between Loads[20]

The analytical alternative is the convolution load flow (CLF). It was proposed by Borkowska in 1974, and applied to the PLF problem successfully[8]. Since then, many studies for solving the PLF problems with mathematical methods have been reported. In most of methods, the output variables are expressed as the linear combination of the input variables by linearization of the load flow equation[7][8][12]-[17]. Also, under the assumption that active loads are independent on reactive one, the probability density function (PDF) or cumulative distribution function (CDF) of each output variable was calculated[8]. However, in large-scaled systems, the CLF requires a huge amount of computational time because all the continuous PDFs of input variables are depicted with discrete points. To obtain the PDFs effectively, the modified methods such as the FFT method[14], the Gram-Charlier methods[3][16], the Cornish-Fisher method[9] were proposed. On the other hand, some methods were presented to consider the correlation between input variables[12][13][16]. However, for simplification of the computation, the linear dependence has been assumed in these methods. Some nonlinear correlation like Fig. 2 should be considered to simulate realistic power system conditions. The figure shows an example of nonlinear correlation between active loads in NYISO.

The stochastic load flow (SLF) is a direct and efficient approach[19]. The method is based on the weighted least squares method that is analogous to static state estimation. It assumes that the nodal specified values follow the Gaussian distribution. In other words, the mean corresponds to the power flow solution obtained by the deterministic power flow calculation. The uncertainty of the specified values may be expressed as the variance of the Gaussian distribution. The difference between the deterministic power flow calculation and this method is to require the additional calculation of the variance. However, since the PDFs of output variables were assumed to be normal distributions, the validity of the technique was discussed[7][13][14].

Since efficient MCS techniques have been developed and the computer performance is significantly improved in recent years, this paper focuses on the MCS-LF method. To develop MCS-LF with the high accuracy, two issues are discussed.

1) How to construct the non-Gaussian distribution model of input variables while preserving the correlation of variables.
2) How to generate the samples from the non-Gaussian distribution while reflecting the correlation.

This paper proposes a new method different to obtain the more accurate PLF solutions. The MLE of PDF of input random variables is estimated by DAEM and the input scenarios are generated by MCMC. To show the performance of the proposed method, the NYISO real data[20] is applied to a sample system in this paper.

II. Maximum Likelihood Estimation of PDF

This section describes the DAEM algorithm that gives the maximum likelihood estimate (MLE) of probability density function (PDF) of input variables. This paper employs DAEM that maintains the non-Gaussianity and the nonlinear correlation of the input variables.

A. Expectation maximization Algorithm

The EM algorithm was presented as a maximum likelihood estimation technique for estimating the probabilistic model from the incomplete data in 1977[21]. It has been successfully applied to estimate the parameters of the mixture distribution model, especially, Multivariate Gaussian Mixture Distribution (MGMD) model in statistical learning, clustering and data communication[22]. The mixture model with M components may be written as

$$p(\boldsymbol{y} \mid \boldsymbol{\theta}) = \sum_{m=1}^{M} \pi_m f_m(\boldsymbol{y} \mid \boldsymbol{\theta}_m) \tag{1}$$

where

$f_m(\boldsymbol{y} \mid \boldsymbol{\theta}_m)$: probability density function conditional on $\boldsymbol{\theta}_m$

π_m: mixing probabilities, $\pi_m \geq 0$ $\sum_{m=1}^{M} \pi_m = 1$

$\boldsymbol{\theta}_m$: parameters of *m-th* component

$\boldsymbol{y}$: incomplete data, $\boldsymbol{y} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_N)$

According to the MGMD, function f may be written as

$$f(\boldsymbol{y} \mid \boldsymbol{\theta}) = f(\boldsymbol{y} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{\frac{p}{2}} |\boldsymbol{\Sigma}|^{\frac{1}{2}}} \exp[-\frac{(\boldsymbol{y}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{y}-\boldsymbol{\mu})}{2}] \tag{2}$$

where

$\boldsymbol{\mu}$: center vector

$\boldsymbol{\Sigma}$: covariance matrix

The log likelihood for is obtained from may be written as

$$L(\boldsymbol{\theta}) = \sum_{n=1}^{N} \log\{\sum_{m=1}^{M} \pi_m f_m(\boldsymbol{y} \mid \boldsymbol{\theta}_m)\} \tag{3}$$

The MLE of $\boldsymbol{\theta}$ is calculated by maximizing (3). The EM algorithm was proposed to solve the nonlinear formulation. In the generalized EM algorithm, $\boldsymbol{y}$ is viewed as an incomplete data associated with latent variables and the EM algorithm maximizes (4) of the conditional expectation of the log likelihood instead of maximizing (3) directly

$$Q(\boldsymbol{\theta} \mid \boldsymbol{\theta}^{(t)}) = E\{\log p(\boldsymbol{y}, \boldsymbol{x} \mid \boldsymbol{\theta}) \mid \boldsymbol{y}, \boldsymbol{\theta}^{(t)}\} = \sum_{n=1}^{N} P(\boldsymbol{x} \mid \boldsymbol{y}, \boldsymbol{\theta}^{(t)}) \log p(\boldsymbol{y}, \boldsymbol{x} \mid \boldsymbol{\theta}) \tag{4}$$

where

$\boldsymbol{y}$: observed variables

$\boldsymbol{x}$: latent variables

$Q(\theta \mid \theta^{(t)})$: conditional expectation of the log likelihood of complete data

$P(\boldsymbol{x} \mid \boldsymbol{y}, \theta^{(t)})$: *posterior* probability of $\boldsymbol{x}$ conditional on $\theta^{(t)}$ and $\boldsymbol{y}$

$\boldsymbol{y}$: current estimate of step t

From the Bayes' theorem, $P(\boldsymbol{x} \mid \boldsymbol{y}, \theta^{(t)})$ may be written as

$$P(\boldsymbol{x} \mid \boldsymbol{y}, \theta^{(t)}) = \frac{P(\boldsymbol{y}, \boldsymbol{x} \mid \theta^{(t)})}{\sum_{n=1}^{N} P(\boldsymbol{y}, \boldsymbol{x} \mid \theta^{(t)})} \tag{5}$$

The EM algorithm may be summarized as follows:

Step 1: Set the initial conditions of $\theta^{(0)}$ and $t \leftarrow 0$

Step 2: (E step) Compute $Q(\theta \mid \theta^{(t)})$

Step 3: (M step) Calculate $\theta^{(t+1)} = \arg\max_{\theta} Q(\theta \mid \theta^{(t)})$

Step 4: Stop if $\left|\frac{\theta^{(t+1)} - \theta^{(t)}}{\theta^{(t)}}\right| \le \varepsilon$. Otherwise, $t \leftarrow t+1$ and return to Step 2

The monotone behavior of the log likelihood function is guaranteed in case where the Q function monotonously increases. If the upper bound of the Q function exists, the EM algorithm converges to local MLE of model parameters. In the case of parameter estimation of MGMD, the Q function may be written as

$$Q(\theta \mid \theta^{(t)}) = \sum_{n=1}^{N} \sum_{m=1}^{M} q_{nm}^{(t)} \log\{\pi_m f_m(\boldsymbol{y}_n \mid \boldsymbol{\theta}_m)\} \tag{6}$$

$$q_{nm}^{(t)} = \frac{\pi_m^{(t)} f_m(\boldsymbol{y}_n \mid \boldsymbol{\theta}_m^{(t)})}{\sum_{m'=1}^{M} \pi_{m'}^{(t)} f_{m'}(\boldsymbol{y}_n \mid \boldsymbol{\theta}_{m'}^{(t)})} \tag{7}$$

where

q_{nm}: *posterior* probability that the observed variable $\boldsymbol{y}_n$ is assigned to the m-th component

Under the constraint conditions of $\sum_{m=1}^{M} \pi_m = 1$, the Q function is evaluated by maximizing (8) with the Lagrange multiplier method.

$$Q(\theta \mid \theta^{(t)}) + \lambda\left(\sum_{m}^{M} \pi_m - 1\right) \tag{8}$$

B. Deterministic Annealing EM Algorithm

The EM algorithm is useful for obtaining a local optimum efficiently. However, it does not necessarily converge to global one due to the influence of initial conditions[22][23]. Therefore, several versions have been proposed to improve the convergence performance to a global optimum[22]-[24]. The DAEM algorithm is known for one of the modified version of EM algorithm in a way that the performance is improved with some simple modification[23]. To avoid local optima, this method smoothes the Q function through introducing the concept of temperature state into the iteration process. From the law of entropy increase, it is repeated until the algorithm converges to an equilibrium configuration for a fixed temperature state while the temperature state slowly decreases

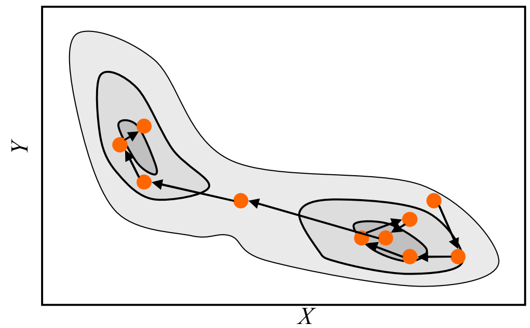

Figure 3. Concept of Metropolis Hastings Sampling

from high to low. In DAEM algorithm, (9) may be rewritten as

$$q_{nm}^{(t)} = \frac{\{\pi_m^{(t)} f_m(\boldsymbol{y}_n \mid \boldsymbol{\theta}_m^{(t)})\}^{\beta}}{\sum_{m'=1}^{M} \{\pi_{m'}^{(t)} f_{m'}(\boldsymbol{y}_n \mid \boldsymbol{\theta}_{m'}^{(t)})\}^{\beta}} \tag{9}$$

β is the temperature parameter that corresponds to reciprocal of temperature. It starts from $\beta_{\min}$ of initial state and ends up to $\beta_{\max}$. If the algorithm converges to each temperate state, β is increased with

$$\beta_{\text{next}} \leftarrow \beta \times \beta_{\text{rate}} \tag{10}$$

III. Metropolis Hastings Sampling

Metropolis Hastings (MH) sampling is known for one of the modified versions of the Metropolis method that carries out sampling process with Markov Chain Monte Carlo[25][26]. To generate the input scenarios while maintaining the non-Gaussianity and the nonlinear correlation, MH sampling is employed to sample from the multi-dimensional PDF of the input variable estimated by DAEM in this paper. The advantages may be summarized as[17][18]

1) It obtains samples from the arbitrary distribution while the correlations between variables are reflected.
2) It is very easy to sample from the multi-dimensional PDF efficiently.
3) The constraints are applied into the algorithm easily so that the evaluation of the extreme value and the truncated distribution becomes possible.
4) The initialization is easy.

The stationary distribution of samples converges to the target distribution while a series of Markov chain sample is generated by the MH sampling. Let us define the probability variables x as $X = \{1, \ldots, m\}^T$. The discrete target distribution $\{\Pi_i\}$ with m class may be written as

$$\Pi_i = \frac{b_i}{C}, \quad i \in X \tag{11}$$

where

$\{b_i\}$: frequency of *i-th* class

C: generalizing constant

If m is large enough, it becomes difficult to apply the traditional Monte Carlo method due to the complex calculation of the generalizing constant[17]. However, there is no need to carry out the calculation in MCMC sampling. The Markov chain $\{\mathbf{x}_t, t = 0,1,\ldots\}$ is generated by obeying the transition matrix $\mathbf{Q} = (q_{ij})$ as follows:

a) When $x_t = i$, the random variable y is generated by obeying $P(y = j) = q_{ij}, \quad j \in X$.

b) When $y = j$, the value of x_{t+1} is accepted according to the acceptance probability α with (12).

$$x_{t+1} = \begin{cases} j & with \quad \alpha_{ij} = \min\left\{\frac{\pi_j q_{ji}}{\pi_i q_{ij}}, 1\right\} = \min\left\{\frac{b_j q_{ji}}{b_i q_{ij}}, 1\right\} \\ i & with \quad 1 - \alpha_{ij} \end{cases} \tag{12}$$

The transition matrix $\mathbf{P} = (p_{ij})$ of next step may be written as

$$p_{ij} = \begin{cases} q_{ij}\alpha_{ij} & if \quad i \neq j \\ 1 - \sum_{k \neq i} q_{ik}\alpha_{ik} & if \quad i = j \end{cases} \tag{13}$$

In case of the continuous PDF $f(x)$, the algorithm of MH sampling can be summarized as

Step 0: Give the current state x_t

Step 1: Generate y from the transition function $q(x, y)$

Step 2: Generate u from U(0,1) and then shift sample x with (14) and (15)

$$x_{t+1} = \begin{cases} y & if \quad u \leq \alpha(x_t, y) \\ x_t & otherwise, \end{cases} \tag{14}$$

$$\alpha(x_t, y) = \min\left\{\frac{f(y)q(y, x_t)}{f(x_t)q(x_t, y)}, 1\right\} \tag{15}$$

The stationary distribution of samples converges to the target distribution while the enough samples are obtained through the iterative computation of Steps 1 and 2.

IV. Proposed Method

This paper proposes a new PLF method for power system operation and planning. The proposed method is based on the hybrid method that consists of DAEM, MH sampling and the AC load flow with the nonlinear equation (see Fig. 4). To consider the non-Gaussianity and the nonlinear correlation of the input variables, this paper develops a new MCS-LF method different from conventional ones. In the proposed method, the MLEs of the multivariate PDF parameters are estimated by the DAEM algorithm from the learning data which is obtained from NYISO[20]. Next, to generate the input scenarios from the PDF estimated, the MH method is employed because it samples from arbitrary distribution. The output scenarios of nodal voltages and line flows are obtained through the AC load flow. Finally, the probabilistic characteristics of output variables are evaluated with the output scenarios. The advantages of proposed may be summarized as

1) The validity of the PDF model is guaranteed due to the MLEs of model parameters.

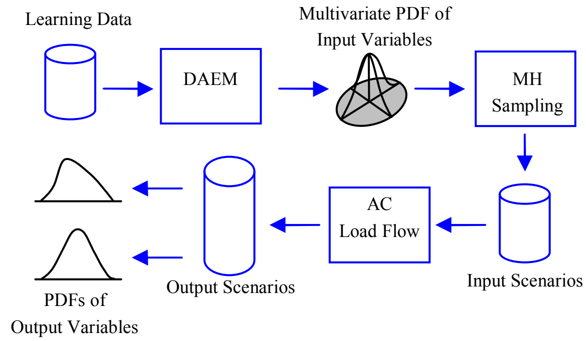

Figure 4. Concept of Proposed Method

2) A complex correlation between input variables is maintained since the nonlinear approximation ability of the MGMD was proven[22][28].

3) The global MLEs of MGMD parameters are obtained because DAEM algorithm is not sensitive to initialization[23].

4) The correlation of PDF estimated is reflected to the input scenarios since MH method are employed[17][18].

5) The extreme value and the truncated distribution is evaluated easily.

There are some assumptions in the proposed method as follow:

a) The probabilistic distributions of all the input variables are expressed as a MGMD model.

b) Neither the branch outage nor the economic dispatch problem is considered.

V. Simulation

A. Simulation Conditions

The proposed method was applied to the Ward & Hale 6 bus system[29] with real load data in NYISO(2005/2/1 a.m.1:00 -2005/3/13 p.m. 4:00, 1000 cases)[20]. In this paper, the learning data of input variables was normalized with (16) and assigned to Table I.

$$Data = Data^* \times \frac{Mean^*}{Mean} \tag{16}$$

where

$Data$: data after normalization

$Data^*$: original data

$Mean^*$: average of original data

$Mean$: average of normalized data according to the designated value of the Ward & Hale 6 bus system

For convenience, the following methods are defined as

Method A: DLF (note, input scenarios are the learning data without modeling and sampling)

Method B: MCS-LF with SRS

TABLE I. SETTING OF INPUT VARIABLES

Input Variables	Mean	Uncertainty [%]	Data Source
$V_{(1)}$	1.05 (fixed)	—	—
$V_{(2)}$	1.1 (fixed)	—	—
$P_{(3)}$	-0.55	13.15	Capital
$P_{(4)}$	0 (fixed)	—	—
$P_{(5)}$	-0.3	10.51	Central
$P_{(6)}$	-0.5	18.90	Dunwoodie
$\delta_{(1)}$	0 (fixed)	—	—
$P_{(2)}$	0.5 (fixed)	—	—
$Q_{(3)}$	0.13	12.50	Genesee
$Q_{(4)}$	0 (fixed)	—	—
$Q_{(5)}$	0.18	12.24	Hudson Valley
$Q_{(6)}$	0.05	15.05	Long Island

Note) $\text{Uncertainty} = \left| \dfrac{Standard \quad Deviation \times 100}{Mean} \right| [\%]$

TABLE II. COMPARISON OF INPUT SCENARIOS WITH METHOD A

Methods	ARMS					
	Mean	Standard Deviation	Kurtosis	Skewness	Correlation Coefficient ($P_{(3)}$-$P_{(5)}$)	Correlation Coefficient ($P_{(3)}$-$P_{(6)}$)
B	0.026%	0.010%	806.982%	6.814%	96.805%	79.524%
C	0.011%	0.105%	19.016%	5.603%	4.362%	8.406%
D	0.022%	0.161%	22.375%	7.247%	3.850%	9.495%

TABLE III. COMPARISON OF OUTPUT SCENARIOS WITH METHOD A

Method	ARMS			
	Mean	Standard Deviation	Kurtosis	Skewness
B	1.035%	12.026%	61.120%	9.323%
C	0.062%	0.251%	15.425%	1.969%
D	0.256%	1.056%	21.328%	2.962%

Method C: MCS-LF with EM and MH

Method D: MCS-LF with DAEM and MH (proposed)

The number of input scenarios of Method A is 1000, and others are 5000. The accuracy of the probabilistic characteristics of output variables is evaluated with the average root mean square (ARMS)[9][15]. Reference [23] is used for the parameter tuning of DAEM.

B. *Simulation Result*

Fig. 5 shows the conditional expectation of the log likelihood of EM and DAEM that are used as parameter estimation of PDF of Methods C and D, respectively. It can be seen that Method C converges to a local optimum due to the high sensitivity to the initial values. On the other hand, Method D provided a global searching to obtain better values while avoiding local optima with smoothing the Q function. It means

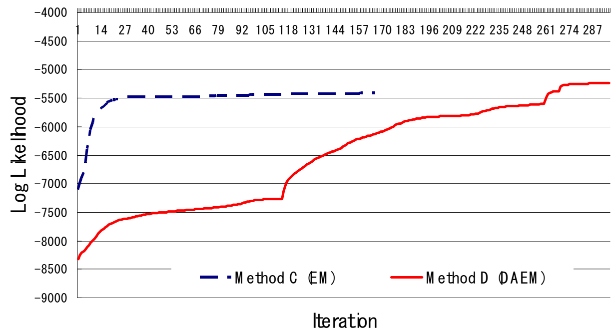

Figure 5. Performance of EM and DAEM

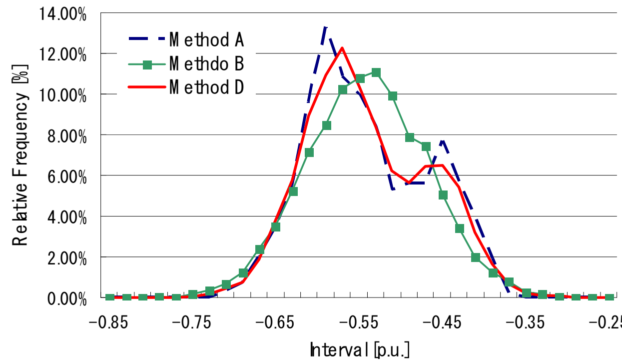

Figure 6. Performance of Non-Gaussian Approximation

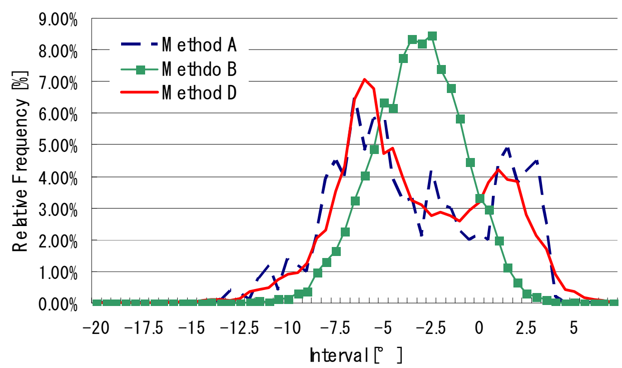

Figure 7. Distribution of Output Variable $\delta_{(2)}$

that the PDFs estimated by Method D are more reasonable than those of Method C. Hereinafter, the comparison between Method A and others are given. The reason is that the result may reflect a part of characteristics of population of output variables because the input scenarios of method are belong to a part of population of input variables. In Table II, the whole probabilistic characteristics of input scenarios are compared. It can be observed that although the mean and standard deviation of input scenarios were good approximated with Methods B, C and D, Method B does not give a good approximation at the kurtosis and the skewness due to the limitation of Method B for the non-Gaussianity. Also, since the skewness of learning data used is small wholly, the performance of the proposed method was clear at Fig. 6. To show the performance of the correlation approximation of the proposed method, this paper gave a example about the correlation between $P_{(3)}$ and $P_{(5)}$ (see Table

II). It can be seen that proposed method generated the input scenarios with extremely high accuracy than the traditional SRS.

Next, the output result is compared in Table III and Fig. 7. In Table III, the wholly probabilistic characters of output scenarios are compared. It can be observed that the accuracy of probabilistic index is better than that of Method B, since the results of proposed method was based on reasonable input scenarios. Also, although the ARMS of the proposed method is slightly larger than that of Method C, the results of the proposed method is more reasonable due to the higher likelihood. Therefore, it turns out that the proposed method obtains the PLF solution with more highly accuracy. Fig. 7 shows the distribution of $\delta_{(2)}$. It can be seen that the results of the proposed method is more actual than that of Method B. Because the number of input scenarios of Method A is limited, the tail of the distribution is intermittent. On the other hand, since good modeling and sampling of input variables were carried out, the proposed method estimated the MLE of the tail. Therefore, it is possible to not only find the potential risks and the weak points of the power system, but also evaluate the impact of uncertainties with the proposed method successfully.

VI. CONCLUSTION

This paper has proposed a new method for the probabilistic load flow calculation. The proposed method makes use of a hybrid method that consists of the DAEM algorithm, the Metropolis Hastings sampling and the AC load flow. The DAEM algorithm plays a key role to estimate the multivariate PDF of input variables from learning with maximizing the likelihood of the PDF so that the non-Gaussianity and the nonlinear correlation of input variables are maintained. The input scenarios of Monte Carlo simulation are generated by the Metropolis Hastings sampling so that the non-Gaussianity and the nonlinear correlation of PDF are reflected. Finally, the output scenarios are computed through a series of the AC load flow. The proposed method was successfully applied to a sample system. The simulation results have shown that the proposed method provides more appropriate PLF solutions than the conventional methods.

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