Capacitor Placement Using Ant Colony Optimization and Gradient

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***Abstract*—This paper aims to minimize the total active losses in electrical distribution systems by means of optimal capacitor bank placement. The proposed methodology to solve this optimization problem is the Ant Colony Optimization (ACO) metaheuristic. The gradient method is combined with the metaheuristic in order to accelerate the convergence of the ACO algorithm. The methodology has been applied successfully in three real systems.**

***Index Terms*—capacitor placement, power summation load flow, ant colony optimization, metaheuristic**

I. Introduction

Since 70s, researchers produce computational methods in order to optimize power systems. Initially, most of the works were applications for systems transmission and using methods based on gradient as a tool for optimization. In the late 80s and early 90s, work oriented to distribution systems began to be more common, especially after the emergence of efficient computational methods, as for example, the power summation load flow method [1] used for simulating electricity distribution radial systems. At that time, also began to emerge general purpose heuristics, called metaheuristics, applied successfully in many optimization problems. For capacitor placement optimization, it is emphasized the Tabu Search [2], [3], Genetic Algorithms [4], [5] and Ant Colony [6], [7], [8], because of the frequency with which they are addressed in technical literature.

This work has as objective the minimization of the active losses in distribution systems, through the placement of capacitor banks. In [9], the solution of this problem was formulated by the gradient method combined with a clustering algorithm that, although fast, does not guarantee to find the global optimal solution. This work combines the gradient method with the ACO metaheuristic. Here, the gradient vector provides a measure of the impact caused by the injection of reactive power losses in the system. The metaheuristic uses this measure to guide and accelerate the search. The result is a methodology able to find more easily the global optimum solution.

This article is organized as follows. In Section II, it describes the problem of optimal capacitor placement. In Section III, it presents a brief introduction to the ACO metaheuristic. In Sections IV and V, it describes the proposed ACO algorithm for capacitor placement. In Section VI, it shows the experimental results of the proposed methodology and discussions of these results. Finally, it presents the conclusions in Section VII.

II. Problem Formulation

The problem of capacitor placement determines the location and sizes of capacitors to meet a predetermined objective, such as minimizing the active losses. Alternatively, it can correct the power factor, improve voltage profile or increase the capacity of transmission lines.

In this work, the function to be optimized is defined as the total active losses of the system. In mathematical terms, the problem is defined as:

$$\text{minimize} \quad L_t = \sum_{i=1}^{n} L_i \tag{1}$$

where n is the number of lines and L_i is the active losses in line i:

$$L_i = \frac{R_i(P_{\Sigma,i}^2 + Q_{\Sigma,i}^2)}{|V_i|^2} \tag{2}$$

where,

- R_i = resistance of the line i;
- L_t = objective function (total active losses);
- $P_{\Sigma,i}$ = active power summation of the downstream system from node i (W);
- $Q_{\Sigma,i}$ = reactive power summation of the downstream system from node i (VAr);
- V_i = voltage at node i (Volts).

It was adopted the simple objective function of Equation (1) in order to focus more on the proposed ACO methodology. Of course, more complex functions, involving constraints, load curves with various levels and demand and loss costs, can be easily incorporated into the objective function.

III. Ant Colony Optimization

A brief description of ACO metaheuristic is given in this section. A complete description is found in [10].

In nature, an ant after finding a food source, it returns to its colony, leaving a trail of pheromone (a chemical) between the food source and its colony. If other ants discover this trail, they will be attracted by the pheromone will follow, with some probability, the same path. Then, these new ants reinforce the pheromone trail. Future ants choose to follow the same trail

initialize
while termination condition not met **do**
  for all ant k **do**
    repeat
      add a bank to a node for ant k by rule of Eq. (3)
    until there is no more improvement in $L_t^{(k)}$, Eq. (1)
    compute $L_t^{(k)}$ by Eq. (1)
    if $L_t^{(k)} < L^*$ **then**
      $\mathbf{x}^* = \mathbf{x}^{(k)}$
      $L^* = L_t^{(k)}$
    end if
  end for
  update the pheromone by Eq. (5) and (6)
end while
return $\mathbf{x}^*$ (the best solution found)

Figure 1. ACO Algorithm

with higher probability, due to the increase of pheromone on the trail. The end result is a strong pheromone scent on the trail connecting the colony to the food source, which attract even more ants. The ACO metaheuristic mimics this behavior of ants to build the solution of search and optimization problems.

Artificial ants implement constructive algorithms. Basically, these algorithms start with a partial solution, and step by step, add a component (e.g., a bank of capacitors) to a partial solution, until finally, to build a complete solution.

At each step, the ant takes a probabilistic decision to choice the next component (capacitor bank) to add to partial solution. This decision depends on two types of information:

1) the *pheromone level* (representing the desirability of a solution component);
2) the *heuristic information* (representing a prior information about the problem).

Immediately after the ant builds a solution, it deposits pheromones on the path that lead the ant to the final solution. This process is repeated until satisfying some stopping criterion (e.g., a maximum number of iterations).

IV. An ACO Algorithm for Capacitor Placement

The followings it is the description of each part of the proposed ACO Algorithm presented in Figure 1.

A. Solution Representation

A solution of a capacitor placement problem is represented by a vector $\mathbf{x}$ of size n. The component x_i of the vector $\mathbf{x}$ represents the number of capacitor banks in node i of the system (e.g., if there are two capacitor banks in node i, then $x_i = 2$).

B. Solution Construction

The ant builds a solution, step by step, as follows. In each step, the ant selects a node and add a capacitor bank to this selected node. The ant stops to add capacitors to nodes, when *there is no more improvement in the objective function value*.

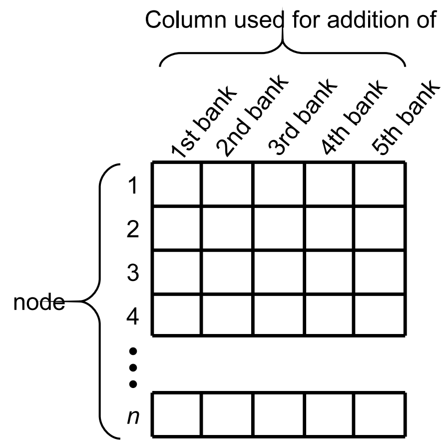

Figure 2. The Pheromone matrix

In ACO metaheuristic, the criterion used by the ant to select a node (in order to add capacitor banks), in each step, is a probabilistic rule involving pheromone and heuristic information. More specifically, an ant k with $x_j^{(k)}$ capacitor banks in node j, has probability p_j of selecting the node j. The value p_j is calculated by the equation:

$$p_j^{(k)} = \frac{(\tau_{j,z_j})^\alpha (\eta_j)^\beta}{\sum_{w=1}^{n} (\tau_{j,z_w})^\alpha (\eta_w)^\beta} \tag{3}$$

where:

- τ is the pheromone matrix (in which is described later);
- η_j is the heuristic information associated with node j (in which is described later);
- α and β are user-defined scaling factors;
- z_j refers to the next bank to be added to node j. So $z_j = x_j + 1$.

The denominator in Equation (3) is a normalization factor.

C. The Pheromone

The pheromone τ is a $n \times 5$ matrix. The row i of the matrix refer to node i and the column j refer to the number of capacitor banks in each node. See Figure 2. Without loss of generality, the maximum number of capacitor banks in a node is limited to the five banks. The matrix element τ_{ij} is a pheromone level representing the desirability of adding the j^{th} capacitor bank to the node i.

D. Heuristic Information

The Heuristic Information is denoted by η_j and represents the desirability of adding a capacitor bank of the node j. In this work, η_j is defined as j^{th} component of gradient vector ∇L_t. That is,

$$\eta_j = \frac{\partial L_t}{\partial Q_{c,j}} \tag{4}$$

where $Q_{c,j}$ is the capacitive reactive power at node j. η_j provides a measure of the impact caused by the injection of reactive power losses in the system. More details in Appendix.

E. Update of Pheromone Trails

After all ants have completed their tasks to add capacitor banks to the nodes, the pheromone trail is updated. Two important events are present in the update: the evaporation and deposit of pheromone. Evaporation reduces the level of pheromone in all elements of the pheromone matrix τ as follows:

$$\tau_{ij} = (1-\rho)\tau_{ij} \quad (5)$$

for $i = 1, \ldots, 5$ and $j = 1, \ldots, n$. ρ is the pheromone evaporation rate in which $0 \leq \rho \leq 1$. The evaporation avoids the level of pheromone grows excessively. So it avoids convergence to suboptimal solutions by providing a mechanism for forgetting the bad decisions of past.

After evaporation, occurs the deposit of pheromone. Ants deposit pheromone in the matrix τ, according to:

$$\tau_{ij} = \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{(k)} \quad (6)$$

for $i = 1, \ldots, 5$ and $j = 1, \ldots, n$, where m is the ant population size. The term $\Delta\tau_{ij}^{(k)}$ is the amount of pheromone that the ant k deposits on the element τ_{ij} and is given by:

$$\Delta\tau_{ij}^{(k)} = \begin{cases} K - L_t^{(k)}, & \text{if } j = x_i^{(k)} \\ 0, & \text{otherwise} \end{cases} . \quad (7)$$

The parameter K is a constant chosen as follows. The value of $K - L_t^{(k)}$ should be positive, so K was defined as the losses of the system with its initial configuration (i.e., without the addition of capacitors). The term $x_i^{(k)}$ is the number of capacitor banks in node i, for the solution found by ant k (see Section IV-A).

V. Implementation of a Memory for Power Flow

The computation of the objective function, Equation (1), requires the expensive computation of a power flow. On the other hand, the ACO algorithm requires a computation of the objective function in each simple step of the solution construction. Consequently, much processing time would be needed to run the ACO algorithm. In order to reduce the processing time, it was implemented a memory to save the N last power flow computations. This approach reduced significantly the processing time.

VI. Experimental Results and Discussion

To evaluate the performance of the ACO algorithm, we used three distribution systems:

- System I - 43 nodes with total load of 2.5 MVA;
- System II - 25 nodes with total load of 1.4 MVA;
- System III - 12 nodes with total load of 1.0 MVA.

The ACO algorithm used populations of 43, 25 and 12 ants, respectively, and a maximum of 30 iterations. For computation of power flow, it used the algorithm given by [1].

In second column of Table I, it shows the value of total losses for the initial configuration (i.e., without the addition of capacitors). The three following columns show losses obtained by ACO algoritm using units of capacitor banks of 100, 200, and 300 kVAr, respectively. If there was margin for reducing losses, the ACO algorithm found a better solution than the initial configuration.

Table I
Total active losses for each system

	Initial	100kVAr	200kVAr	300kVAr
System I	0.6795	0.5748	0,6795	0.6795
System II	4.3029	3.5364	3,6225	4.3029
System III	17.1000	15.2822	15,4528	15.5393

Table II
Number of added capacitor banks

	100kVAr	200kVAr	300kVAr
System I	1	0	0
System II	2	1	0
System III	4	2	1

Table III
Number of iterations to find the best solution

	100kVAr	200kVAr	300kVAr
System I	5	0	0
System II	4	1	0
System III	8	4	1

Table II presents the number of capacitor banks placed in each system. These results show that the ACO algorithm worked correctly in its task of placement because it not added capacitors of 200 MVA and 300 MVA to the system I, since these capacitors have high powers for the system I (its addition would increase the losses).

In Table III, it is shown the number of iterations performed by ACO algorithm for obtaining the best solution. In all cases simulated, the solution was found with few iterations.

In Table IV shows an different experiment to study the influence of the gradient, in which is used as heuristic information of the ACO algorithm, according to Equation 4. Here, the algorithm was applied to the systems using two distinct approaches: with information heuristic (i.e., gradient and pheromone) and without information heuristic (i.e., only pheromone). Table IV shows the number of iterations to find the optimal solution. In all cases, the number of iterations was lower using the information heuristic.

In Table IV, the system I was simulated using banks of 10 kVAr, whereas systems II and III used banks of 50 kvar. These results show that the gradient was efficient for performing the role of the information heuristic of ACO algorithm.

VII. Conclusion

The ACO metaheuristic application for the capacitor placement problem was efficient to find the optimal solution for the systems used in this work. The proposal presented here, using the gradient vector as heuristic information to the colony

Table IV
COMPARISON WITH AND WITHOUT INFORMATION HEURISTIC

	with	without
System I	25	30
System II	12	15
System III	15	19

of ants, was effective and promising in the experiments. Moreover, the work proposed an ACO algorithm with some innovations, such as the method to construct solutions and structure of the pheromone matrix. Researchers are underway in order to adapt the ACO algorithm for using unit of capacitors of variable sizes.

APPENDIX A

This appendix presents a detailed version of Equation 4 as follows:

$$\eta_j = \frac{\partial L_t}{\partial Q_{c,j}} \tag{8}$$

$$= \sum_{i=1}^{n} \frac{\partial L_i}{\partial Q_{c,j}} \tag{9}$$

where,

$$\begin{aligned} \frac{\partial L_i}{\partial Q_{c,j}} &= \frac{\partial}{\partial Q_{c,j}} \left[\frac{R_i(P_{\Sigma,i}^2 + Q_{\Sigma,i}^2)}{|V_i|^2} \right] \\ &= \frac{2R_i}{V_i^2} \left(P_{\Sigma,i} \frac{\partial P_{\Sigma,i}}{\partial Q_{c,j}} + Q_{\Sigma,i} \frac{\partial Q_{\Sigma,i}}{\partial Q_{c,j}} \right) \\ &\quad - \frac{\partial V_i^2}{\partial Q_{c,j}} \left(\frac{P_{\Sigma,i}^2 + Q_{\Sigma,i}^2}{V_i^4} \right) \end{aligned} \tag{10}$$

and,

$$\begin{aligned} \frac{\partial V_i^2}{\partial Q_{c,j}} &= \frac{1}{2} \Bigg\{ -2X_i \frac{\partial Q_{\Sigma,j}}{\partial Q_{c,j}} + \frac{\partial V_k^2}{\partial Q_{c,j}} \\ &\quad + \frac{1}{2}(B^2 - 4C)^{-1/2} \\ &\quad \times \left[2B \left(2X_i \frac{\partial Q_{\Sigma,i}}{\partial Q_{c,j}} - \frac{\partial V_i^2}{\partial Q_{c,j}} \right) \right. \\ &\quad \left. - 8(R_i^2 + X_i^2) Q_{\Sigma,i} \frac{\partial Q_{\Sigma,i}}{\partial Q_{c,j}} \right] \Bigg\} \end{aligned} \tag{11}$$

where,

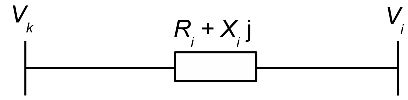

Figure 3. A line of the system

$$B = 2(R_i P_{\Sigma,i} + X_i Q_{\Sigma,i}) - V_i^2 \tag{12}$$

$$C = (R_i^2 + X_i^2)(P_{\Sigma,i}^2 + Q_{\Sigma,i}^2) \tag{13}$$

Some terms used here are explained in Figure 3. It is worth noting that for the slack bus ($i = 0$), has

$$\frac{\partial V_0^2}{\partial Q_{c,j}} = 0 \tag{14}$$

because V_0 is constant. For more information, see [9].

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REFERENCES

[1] G. R. Cespedes, "New method for analysis of distribution networks," *IEEE Transactions on Power Delivery*, vol. 5, no. 1, 1990.

[2] R. A. Gallego, A. J. Monticelli, and R. Romero, "Optimal capacitor placement in radial distribution networks," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 630–637, 2001.

[3] Y.-C. Huang, H.-T. Yang, and C.-L. Huang, "Solving the capacitor placement problem in a radial distribution system using tabu search approach," *IEEE Transactions on Power Systems*, vol. 11, no. 4, pp. 868–873, 1996.

[4] T. Ghose, S. Goswami, and S. Basu, "Energy loss reduction in distribution system by capacitor placement through combined ga-sa technique," in *TENCON '98 - IEEE Region 10 International Conference on Global Connectivity in Energy, Computer, Communication and Control*, vol. 2, 1998, pp. 502–505.

[5] K.-H. Kim and S.-K. You, "Voltage profile improvement by capacitor placement and control in unbalanced distribution systems using ga," in *IEEE Power Engineering Society Summer Meeting 1999*, vol. 2, 1999, pp. 800–805.

[6] R. Annaluru, S. Das, and A. Pahwa, "Multilevel ant colony algorithm for optimal placement of capacitors in distribution systems," in *Congress on Evolutionary Computation CEC2004*, 2004.

[7] P. Gardel, B. Baran, H. Estigarribia, U. Fernandez, and S. Duarte, "Multiobjective reactive power compensation with an ant colony optimization algorithm," in *The 8th IEE International Conference on AC and DC Power Transmission, 2006. ACDC 2006*, 2006.

[8] C.-F. Chang, "Reconfiguration and capacitor placement for loss reduction of distribution systems by ant colony search algorithm," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1747–1755, 2008.

[9] M. F. M. Junior and M. C. P. Filho, "Optimal power flow in distribution networks by newton' s optimization methods," in *Proceedings of the 1998 IEEE International Symposium on Circuits and Systems*, Montrey, CA, 1998.

[10] M. Dorigo and T. Stützle, *Ant Colony Optimization*. MIT Press, 2004.