A Game Theoretic Framework for Generation Maintenance Scheduling in Oligopolistic Electricity Markets

M. A. Fotouhi and S. M. Moghaddas Tafreshi

***Abstract*—This paper studies the maintenance decisions of generating companies (GENCOs) which are fully engaged in oligopolistic electricity market. Maintenance decisions in an oligopolistic electricity market have a strategic function, because GENCOs usually have impacts on market prices through capacity outages. The main contribution of this paper is modeling a game theoretic framework to analyze strategic behaviors of GENCOs. Each GENCO tries to maximize its payoff by strategically making decisions, taking into account its rival GENCOs' decisions. Cournot-Nash equilibrium is used for decision making on maintenance problem in Oligopolistic electricity market. The analytic framework presented in this paper enables joint assessment of maintenance and generation strategies; it also considers the regulation of ISO on GENCOs' desired maintenance plan.**

***Index Terms*—Cournot-Nash equilibrium, game theory, maintenance scheduling of generating units, Oligopolistic market.**

I. Nomenclature

A. Variables

$g_{ij}(t)$	Power generated by unit *j* of GENCO*i* in period *t* (MW).
$G(t)$	Total energy supply in period *t* (MWh).
i	GENCO.
j	Generating unit.
$m_{ij}(t)$	Binary maintenance decision variable for unit *j* of GENCO*i* in period *t* (1 if unit j is on maintenance in period *t* and 0 otherwise).
t	Time periods (week).
T	Number of time periods (52).

B. Constants

$\gamma(t)$	Intercept of the linear price/demand curve ($/MWh).
$\lambda(t)$	Slope of the linear price/demand curve $(\$/(\text{MWh})^2)$.
$D(t)$	Power demanded in period *t* (MW).
FOR_{ij}	Forced outage rate of unit *j* of GENCO*i*.
G_i	Set of indices of generating units owned by GENCO*i*.
$h(t)$	Hours of period *t* (168 hours).
I	Number of GENCOs.
MC_{ij}	Maintenance cost of unit *j* of GENCO*i* ($/MW).
M_i	Maximum numbers of units in maintenance for GENCO*i*.
N_{ij}	Duration of the maintenance outage of unit *j* of GENCO*i*.
$P_{ij}^{\max}$	Capacity of unit *j* of GENCO*i*.
PC_{ij}	Production cost of unit *j* of GENCO*i* ($/MWh).
$R(t)$	Reserve requirement in period *t*.

M. A. Fotouhi is with K. N. Toosi University of Technology, Tehran, Iran (e-mail: fotouhi@ee.kntu.ac.ir).

S. M. Moghaddas Tafreshi is with K. N. Toosi University of Technology, Tehran, Iran (e-mail: tafreshi@eetd.kntu.ac.ir).

II. Introduction

PREVENTIVE maintenance scheduling of generating units is an important mission in power system and plays vital role in operation and planning of the system. Maintenance strategy is a complicated optimization problem for mid-term power systems operations planning. Several methods have been proposed recently to solve the maintenance scheduling of generating units. It is in fact determining the maintenance period of time for individual generators subject to several constraints over a given horizon.

In a centralized electric power system, an appropriate generation maintenance scheduling is derived by the system operator and imposed to producers [1]. The reliability evaluation of maintenance scheduling and least-cost optimization algorithms had been one of the main concerns for the last few decades [2]. In most of the previous works, however, generators are maintained or not for system-wide reliability or least-cost rather than for their own profitability. In the new competitive environment, customers request for high reliability services with lower electricity prices, while GENCOs have to make their own profit.

In a competitive market environment which the management of GENCO and ISO is separated, unit maintenance scheduling is determined through multiple interactions between ISO and GENCOs each maximizing its own benefit. GENCOs will try to schedule their units for maintenance in order to maximize their benefit. The ISO seeks a generation maintenance annual plan that ensures similar reliability through the weeks of the year, prior to the ISO's coordination process, individual GENCOs should have their own maintenance strategies in advance [3].

In deregulated power systems usually GENCOs have independence to maintain their generators in a decentralized manner [4]. Strategic behaviors of GENCOs can be modeled in a game theoretic framework, and players of the game correspond to GENCOs.

Maintenance of generating units may cause outage of a significant amount of its capacity. Withdrawing generation capacity is a strategic decision GENCOs adopt in order to increase electricity prices. Maintenance decisions is a way of withdrawing some parts of generation capacity for a period of time, therefore, it has potentially major impacts on spot prices [4].

In this study, a game theoretic framework is suggested to solve maintenance scheduling of generating units under competitive market environment. Gaming considerations can earn GENCOs higher profit because they can effectively exercise market power when their competitors' capacity is on maintenance [4].

Most of the research done on the electricity markets was based on the Cournot model. The Cournot oligopoly model assumes that each strategic firm decides its quantity to produce, while treating the output level of its competitors as a constant. The optimal strategy profile is defined by Cournot-Nash equilibrium of the game.

The regulation of ISO is also considered, and the difference between their scheduling is compared. The objective in using such a method is to attain an applicable blend between maximum reliability of the system and maximum benefit of generating firms. Note that maintenance outages decrease reliability and increase operation cost. Although the coordination process how ISO adjusts the individual GENCOs' maintenance schedules and how each GENCO responds to the ISO's proposed schedule is important, it is beyond the scope of this paper.

A hypothetical test system is considered to show the applicability of the proposed model. The results obtained point out that maintenance scheduling can be one of the important strategic behaviors whereby GENCOs maximize their profit in a competitive market environment.

III. Cournot Behavior of GENCOs

In order to analyze real markets, economists have developed models between two extreme cases, perfect competition and pure monopoly. Oligopoly competition refers to a market structure where a few players coexist. Taking perfect competition and monopoly models as the end points, there is an infinite number of theoretical possibilities for oligopoly models, all of which differ mainly in the assumptions used to characterize market structure and firm interdependencies.

The Cournot model has become a classic in microeconomic oligopoly theory. Cournot games have the following characteristics in common:

- competition occurs only in quantities
- product is homogeneous
- market price is determined by auction
- Players schedule for their production simultaneously.

In the Cournot model, each firm chooses an output quantity to maximize profit. It is assumed that quantities produced are immediately sold. Market price in the model is determined through an auction process that equates industry supply with aggregate demand. The model also assumes that all firms in the industry can be identified at the start of the game, and that decision-making by firms occurs simultaneously [5].

Each firm is sufficiently large to influence market price received by all, and the quantity produced by other firms. Each firm maximizes its own profit given the quantity chosen by other firms expressed as [6].

$$\pi^i(q_i, q_{i'}) = q_i \cdot P(q_i, q_{i'}) - C_i(q_i)\,. \qquad (1)$$

$\pi^i(\cdot)$	Profit of player given the production strategy of all other players i';
q_i	Production strategy of player *i*;
$P(\cdot)$	Price as a function of all *q*'s i.e., firms *i*, *i'*, etc. This is the main characteristic of an oligopolistic market that distinguishes such markets from a perfectly competitive one—players can influence market price by changing their production as opposed to a "price taker" behavior exhibited in a competitive market;
$C_i(\cdot)$	Cost as a function of production strategy q_i.
Q	Aggregate demand.
I	Number of players.

The solution of the game is obtained by solving a set of simultaneous equations representing the first order optimality conditions for each firm *i* [6].

The Nash equilibrium formation of Cournot's duopoly model is shown in Fig. 1. The two axes define the output of the firms. In this model the reaction curve represents how much each firm would produce given the production decision from the other firm. The Nash equilibrium defines where each firm has maximized profit, given the output of the other.

The Cournot model is often used to describe the behavior of generating companies (GENCOs) in electricity markets. We consider n GENCOs, assume that each GENCO uses a Cournot model to derive its generation and maintenance scheduling in the market, and obtain some relevant analytical results characterizing the market. GENCOs are assumed to maximize profit according to the Cournot assumption using production quantities as the decision variable. It is considered that the transmission network does not influence the equilibrium, i.e., that no network constraint is binding. Cournot models are often encountered in the technical literature as they adequately represent producer behavior in real-world markets [7].

A Cournot-Nash equilibrium outcome is characterized by the familiar Nash equilibrium concept that no player can gain any additional profit by changing its own generation strategy while every other player keeps its own generation unchanged [4].

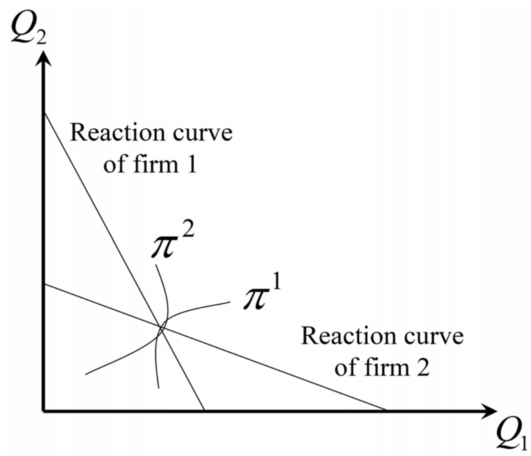

Fig. 1. Nash-Cournot Equilibrium in duopoly model.

IV. Linear Demand Equation

In characterizing market participants, producers are usually represented by their cost curves and consumers by their demand curves. It is relatively easier to quantify cost curves, for example producers with thermal generators are often modeled by their heat rate and fuel cost. In comparison, demand curves are more difficult to empirically quantify as they are derived from the subjective utility of consuming volumes electricity [8]. If the GENCOs do not know the linear demand function, they must estimate the demand. Different methods have been proposed for estimating linear demand function which is not a main concern of this paper.

It is generally difficult to determine demand curves from fundamental utility considerations as utility is intrinsically an ordinal quantity, also the consumer income constraints required for the derivation are largely unknown. Consequently with few exceptions, most market simulations model demand by directly specifying a representative demand function which relates the price a consumer is prepared to pay for a volume of electricity [8].

If it is assumed that there is negligible transmission loss, then the aggregate demand Q will be equal to the total output of all the Gencos in the market as

$$Q = \sum_{i=1}^{I} q_i \,. \tag{2}$$

The market price P depends on Q and their relationship is represented by the inverse linear demand function in (3)

$$P = \gamma - \lambda \cdot Q \,. \tag{3}$$

Where γ, λ are the positive coefficients of the linear demand function [9].

V. Firms' Scheduling

GENCOs would try to extract maximum profit by scheduling their maintenance in a period to incur least opportunity costs, and to make the most of all other periods when its competitors' generators are scheduled for maintenance [4]. Note that all GENCOs considered are price-maker, i.e., they have the capability of altering market clearing prices. We may think of each GENCO trying to work out the best generation strategy as well as the maintenance decisions by looking at the mutual impact of maintenance on their base reaction function [4]. GENCOs' objective for maximization is

$$\begin{aligned} Max\ \Pi_i = & \sum_{t=1}^{T} \sum_{j \in G_i} \left(\gamma(t) - \lambda(t) \cdot G(t)\right) \cdot G(t) \\ & - \sum_{t=1}^{T} \sum_{j \in G_i} P_{ij}^{\max} \cdot MC_{ij} \cdot m_{ij}(t) \\ & - \sum_{t=1}^{T} \sum_{j \in G_i} g_{ij}(t) \cdot PC_{ij} \cdot h(t) \end{aligned} \tag{4}$$

Subject to $$h(t) \cdot \sum_{i=1}^{I} \sum_{j \in G_i} g_{ij}(t) = G(t) \,. \tag{5}$$

For the purpose of simplicity, no uncertainty is considered, which means that appropriate linear demand functions are forecasted for each week.

The set of constraints of the maintenance scheduling problem of the GENCO*i* are defined below.

1) Maintenance Outage Duration: The following constraint guarantees for each unit that it is maintained the required number of time periods [10]

$$\sum_{t=1}^{T} m_{ij}(t) = N_{ij} \qquad \forall i, \forall j \in G_i \,. \tag{6}$$

2) Continuous Maintenance: The constraint below ensures that the maintenance of any unit must be completed once it begins [10]

$$m_{ij}(t) - m_{ij}(t-1) \le m_{ij}(t + N_{ij} - 1) \quad \forall i, \forall t, \forall j \in G_i \tag{7}$$

3) Maximum Number of Units Simultaneously in maintenance: Constraint (10) limits the maximum number of units that GENCO*i* can maintain at the same time [1]

$$\sum_{j \in G_i} m_{ij}(t) \le M_i \qquad \forall i, \forall t_i \,. \tag{8}$$

4) Maintenance Exclusion: This constraint enforces the impossibility of maintaining two prespecified unit of the same GENCO at the same time [10]

$$m_{ij}(t) + m_{ij'}(t) \le 1 \qquad \forall i, \forall t.. \tag{9}$$

It means parallel maintenance of unit j and j' of GENCO*i* is impossible.

VI. Regulation by ISO

It should be noted that the role of ISO is to ensure system security. Therefore, it must agree with GENCOs on a generation maintenance plan that preserves system security. The ISO solves a maintenance scheduling problem involving all units, independently on which GENCO owns each unit, with the target of maximizing the reliability throughout the weeks of the year. Sufficiently accurate demand forecasts for the whole year are considered known [10].

The ISO compares the maintenance outage timing scheduled by GENCOs with its desired timing. If the reliability index of two plans are closed to each other, the ISO accepts the GENCO's decision, If not ISO encourages GENCOs to alter their plan by proposing incentives. The objective function of ISO can be formulated as follow

$$Max \ \frac{1}{T}\sum_{t=1}^{T}\frac{\left[\sum_{i=1}^{I}\sum_{j\in G_i}P_{ij}^{\max}\cdot\left(1-m_{ij}(t)\right)\cdot\left(1-FOR_{ij}\right)\right]-D(t)}{\left[\sum_{i=1}^{I}\sum_{j\in G_i}P_{ij}^{\max}\cdot\left(1-FOR_{ij}\right)\right]-D(t)} \quad (10)$$

The ISO considers the constraints mentioned for generating units. It also has a constraint which guarantees a net reserve above a specified threshold for all periods [1]

$$\sum_{i=1}^{I}\sum_{j\in G_i}\left[P_{ij}^{\max}\cdot\left(1-m_{ij}(t)\right)\cdot\left(1-FOR_{ij}\right)\right]-D(t)\geq R(t)\cdot \quad (11)$$

VII. Case Study

A hypothetical system formed by 11 generating units belonging to 3 GENCOs is introduced (Table I). The specific data of generating units is based on IEEE reliability test system [11]. GENCOs' individual limitations shown in Table II. depend to there structure and desires.

TABLE I
TEST SYSTEM

GENCOs	Units	Capacity (MW)	Maintenance Duration (weeks)	Production Cost ($/MWh)	Maintenance Cost ($/MWh)
	u1	400	8	40	30
GENCO1	u2	197	6	44	27
	u3	197	6	38	27
	u4	76	3	31	21
	u5	350	8	42	32
GENCO2	u6	155	5	28	25
	u7	100	4	34	33
	u8	20	1	35	18
	u9	400	8	40	35
GENCO3	u10	350	8	42	32
	u11	12	1	31	19

TABLE II
GENCOS' LIMITATIONS

	GENCO1	GENCO2	GENCO3
Maintenance exclusion z	(u1,u2) (u1,u3)	(u5,u6) (u5,u7)	(u9,u10)
Maximum number of units on maintenance at t	2	3	2

In Fig. 2 the peak demand and average demand during 52 weeks of the year is shown. The weekly electricity price and demand of Omel deregulated power system is used and they are normalized to match our hypothetical system [12]. Linear demand function parameters are actually captured from loads, but for the sake of simplicity they are estimated from demand and price data.

Table III illustrates GENCOs initial maintenance plan which should be checked by ISO. In time periods not shown in the table no maintenance outage is anticipated by GENCOs. The model is implemented using GAMS.

ISO has its own maintenance plan, hence GENCOs send their initial plan to ISO and it will be compared with ISO's desired plan. Table IV illustrates ISO's ideal plan.

TABLE III
GENCOS' INITIAL MAINTENANCE PLAN

	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11
t1	0	1	1	0	0	0	0	0	0	0	0
t2	0	1	1	0	0	0	0	0	0	0	0
t3	0	1	1	0	0	0	0	0	0	0	0
t4	0	1	1	0	0	0	0	0	0	0	0
t5	0	1	1	0	0	0	0	0	0	0	0
t6	0	1	1	0	0	0	0	0	0	0	0
t11	0	0	0	0	0	0	1	0	0	0	0
t12	0	0	0	0	0	0	1	0	1	0	0
t13	0	0	0	0	0	0	1	0	1	0	0
t14	0	0	0	0	0	0	1	0	1	0	0
t15	0	0	0	0	0	1	0	0	1	0	0
t16	0	0	0	0	0	1	0	0	1	0	0
t17	0	0	0	0	0	1	0	0	1	0	0
t18	0	0	0	0	0	1	0	0	1	0	0
t19	0	0	0	0	0	1	0	0	1	0	0
t28	0	0	0	1	0	0	0	0	0	0	0
t29	0	0	0	1	0	0	0	0	0	0	0
t30	0	0	0	1	0	0	0	0	0	0	0
t32	0	0	0	0	0	0	0	1	0	0	0
t36	0	0	0	0	0	0	0	0	0	1	0
t37	0	0	0	0	0	0	0	0	0	1	0
t38	0	0	0	0	0	0	0	0	0	1	0
t39	0	0	0	0	0	0	0	0	0	1	0
t40	0	0	0	0	0	0	0	0	0	1	0
t41	0	0	0	0	0	0	0	0	0	1	0
t42	0	0	0	0	0	0	0	0	0	1	0
t43	0	0	0	0	0	0	0	0	0	1	0
t44	1	0	0	0	0	0	0	0	0	0	0
t45	1	0	0	0	1	0	0	0	0	0	0
t46	1	0	0	0	1	0	0	0	0	0	0
t47	1	0	0	0	1	0	0	0	0	0	0
t48	1	0	0	0	1	0	0	0	0	0	0
t49	1	0	0	0	1	0	0	0	0	0	0
t50	1	0	0	0	1	0	0	0	0	0	0
t51	1	0	0	0	1	0	0	0	0	0	0
t52	0	0	0	0	1	0	0	0	0	0	1

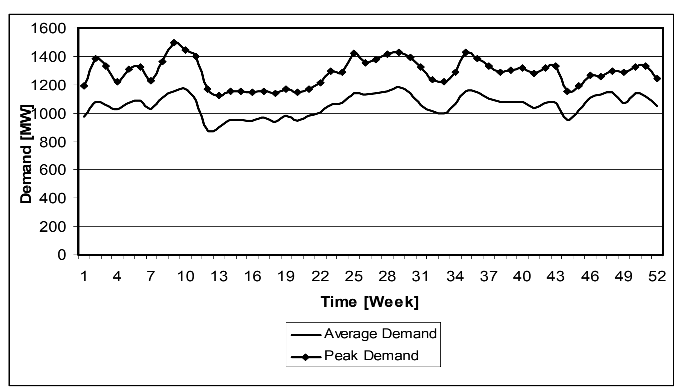

Fig. 2. Demand graph during 52 weeks.

In Fig. 3 the difference between maintenance scheduling of GENCOs and ISO's ideal plan is shown. Now it needs coordination between them to achieve the final plan.

TABLE III
ISO'S IDEAL MAINTENANCE PLAN

	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11
t1	1	0	0	0	0	0	0	1	0	0	1
t2	1	0	0	0	0	0	0	0	0	0	0
t3	1	0	0	0	0	0	0	0	0	0	0
t4	1	0	0	1	0	0	0	0	0	0	0
t5	1	0	0	1	0	0	0	0	0	0	0
t6	1	0	0	1	0	0	0	0	0	0	0
t7	1	0	0	0	0	0	0	0	0	0	0
t8	1	0	0	0	0	0	0	0	0	0	0
t11	0	1	0	0	0	0	0	0	0	0	0
t12	0	1	0	0	0	1	0	0	0	0	0
t13	0	1	0	0	0	1	0	0	0	0	0
t14	0	1	0	0	0	1	0	0	0	0	0
t15	0	1	0	0	0	1	0	0	0	0	0
t16	0	1	0	0	0	1	0	0	1	0	0
t17	0	0	0	0	0	0	0	0	1	0	0
t18	0	0	0	0	0	0	0	0	1	0	0
t19	0	0	0	0	0	0	0	0	1	0	0
t20	0	0	0	0	0	0	1	0	1	0	0
t21	0	0	0	0	0	0	1	0	1	0	0
t22	0	0	0	0	0	0	1	0	1	0	0
t23	0	0	0	0	0	0	1	0	1	0	0
t24	0	0	0	0	1	0	0	0	0	0	0
t25	0	0	0	0	1	0	0	0	0	0	0
t26	0	0	0	0	1	0	0	0	0	0	0
t27	0	0	0	0	1	0	0	0	0	0	0
t28	0	0	0	0	1	0	0	0	0	0	0
t29	0	0	0	0	1	0	0	0	0	0	0
t30	0	0	0	0	1	0	0	0	0	0	0
t31	0	0	0	0	1	0	0	0	0	0	0
t38	0	0	0	0	0	0	0	0	0	1	0
t39	0	0	0	0	0	0	0	0	0	1	0
t40	0	0	0	0	0	0	0	0	0	1	0
t41	0	0	0	0	0	0	0	0	0	1	0
t42	0	0	0	0	0	0	0	0	0	1	0
t43	0	0	0	0	0	0	0	0	0	1	0
t44	0	0	0	0	0	0	0	0	0	1	0
t45	0	0	1	0	0	0	0	0	0	1	0
t46	0	0	1	0	0	0	0	0	0	0	0
t47	0	0	1	0	0	0	0	0	0	0	0
t48	0	0	1	0	0	0	0	0	0	0	0
t49	0	0	1	0	0	0	0	0	0	0	0
t50	0	0	1	0	0	0	0	0	0	0	0

VIII. Conclusion

The proposed framework is simple to implement in practice, and requires a reasonably small amount of computing time and a small amount of data communication. It is presented based on the fact that GENCOs will try to strategically place their maintenance and dispatch their generators taking into consideration their rivals' strategies. This implicitly assumes each GENCO is able to at least "guess" the capacity and cost parameters of its competitor GENCOs—an assumption that is not unreasonable or unrealistic.

The results emphasize the need to an advanced coordination procedure between GENCOs and ISO in order to acquire final maintenance scheduling plan.

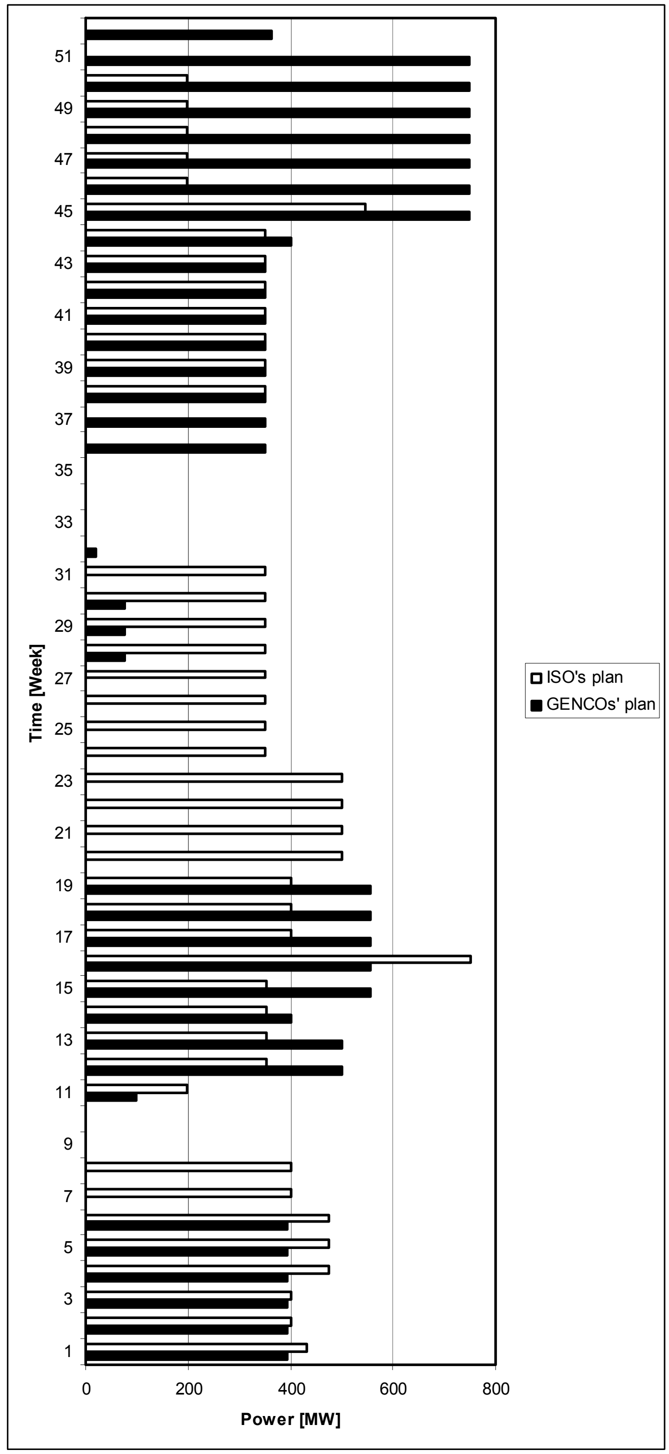

Fig. 3. Different plans of GENCOs and ISO.

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