# Comparison of Enhanced-PSO and Classical Optimization Methods: a case study for STATCOM placement

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***Abstract*—This paper validates the effectiveness of an enhanced particle swarm optimizer (Enhanced-PSO) method in solving the problem of optimal allocation of FACTS devices in a power system. The performance of the Enhanced-PSO method is compared with classical optimization approaches using a simple but realistic case study of optimal allocation of STATCOM devices, considering steady state and economic criteria. This paper also discusses the concepts and details about the optimization process that tend to be overlooked in the literature since the selection of an optimization algorithm highly depends on them.**

***Index Terms*— FACTS devices, classical optimization, Benders' decomposition, branch and bound, evolutionary computation techniques, particle swarm optimization.***

## I. Introduction

THE topic of optimal allocation of FACTS (Flexible AC Transmission System) devices is still in a relatively early stage of investigation. Currently, there is no widely accepted method and many researchers claim their methods to be "better" than others. Considering the present state-of-the-art in this area, a comparison of different methods, particularly between classical and metaheuristic approaches, has been difficult because each study focuses on different problem formulations, system sizes and operating conditions.

This paper provides a common background for comparing the performance of classical and metaheuristic optimization algorithms. In particular between Bender's decomposition and Branch-and-bound (B&B) and the Enhanced-PSO method, which has been proven to be more effective than other metaheuristic optimization techniques [1]. A simple but realistic case study of optimal STATCOM allocation (a type of FACTS device), based on steady state and economic criteria, is used as an illustrative example.

It is important to note that the focus of this paper is not to find a solution to the particular problem, but rather to illustrate the differences between classical deterministic approaches and metaheuristic techniques and to comment on important details about the optimization process that tend to be overlooked in the literature: optimization of offline problems, understanding convexity assumptions (that do not apply only to the objective function), and the discussion about local versus global optimality (for a given objective function).

The following sections of this paper provide: an optimization background (section II), concepts and issues that should not be disregarded (section III), a problem description (section IV), optimization algorithms (section V), simulation results (section VI), and concluding remarks (section VII).

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## II. Background

The optimization techniques used to solve the optimal allocation of FACTS devices can be decomposed into two primary groups: classical approaches and metaheuristic algorithms, consisting of mainly evolutionary computation techniques (ECTs). A third group of alternative methods, such as modal analysis, may also be considered. However, these methods are primarily based on technical feasibility rather than on finding optimal solutions.

### *A. Classical Optimization Techniques*

In the literature, two classes of classical optimization methodologies have been applied to this problem: (i) Mixed Integer Linear Programming (MILP) [2]-[4] and (ii) Mixed Integer Non-Linear Programming (MINLP) [5]-[7].

On the one hand, the MILP formulation, as the name indicates, requires the relationships between all variables to be linear. Thus, this approach can be only used together with DC power flow only. The main algorithms for solving the MILP problem are Bender's Decomposition [2], Branch and Bound (B&B), and Gomory cuts [3], [4]. On the other hand, the MINLP formulation allows for the use of a non-linear objective function and constraints, thus, AC power flow can be used in this case. The algorithm most widely utilized for solving the MINLP problem is Bender's Decomposition [5]-[7]. Unfortunately, the size and non-convexity of the problem, which depend on the system parameters, are critical issues that may cause convergence problems.

### *B. Metaheuristic Techniques*

Computational intelligence based techniques, such as Genetic Algorithm (GA) [5], [6], [8]-[11], Particle Swarm

Optimization (PSO) [12]-[14], Simulated Annealing (SA) [8], [15], Tabu Search (TS) [14], [15], and Evolutionary Programming (EP) [16], [17], are alternative methods for solving complex optimization problems.

Candidate solutions play the role of individuals in a population and the cost function determines the environment where the solutions exist. Evolution of the population then takes place and, after the repeated application of biological or social operators, the optimal solution is reached.

In general ECTs perform well in MINLP problems. However the scalability of these methods requires further investigation.

## III. Optimization: Concepts and Issues

### *A. Optimization of offline problems*

A common misperception is the belief that the problem of optimal allocation of FACTS devices is not challenging from the optimization perspective because it is an offline problem. Some presume that the solution is as simple as arranging a number of computers in parallel and letting them run, for as long as it takes, until all possible solutions are found and the best one selected among them.

The fact is that, even when this approach is theoretically possible to perform for any system, in practice the number of calculations required to find the solutions to the problem can grow extremely fast as the size of the system increases and the objective function becomes more sophisticated (evaluation of transient performance is computationally intensive). If it is also required to satisfy the N-1 or N-2 contingency criteria, or add stochastic components and uncertainties to the system, the number of cases to evaluate simply becomes uncountable.

Therefore, the study of optimization algorithms applied to system planning problems, such as the problem of FACTS allocation is not trivial.

### *B. Convexity assumptions*

The concept of convexity is mostly analyzed in the case of the objective function: if the function is strictly convex a unique optimal solution is guaranteed (Fig. 1.a).

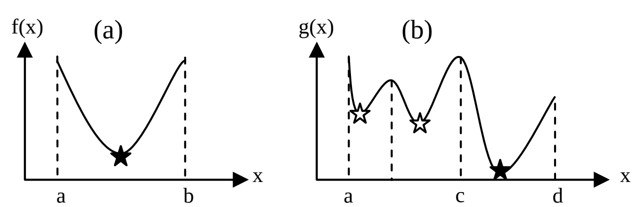


Fig. 1: Global versus local minima

This characteristic is most desirable but it rarely occurs in power system problems. Most of the time the plot of the objective function resembles the function in Fig. 1.b. As a result, gradient descent algorithms are prone to getting trapped in local valleys (local minima). In these cases, special mechanisms, such as injecting randomness to the search, must be considered.

The convexity assumption also applies to the feasible region. For example, in the case of linear programming problems, the optimum can be found (either by simplex method or interior point method) if the feasible region is a convex set, as shown in Fig. 2.a (as opposed to Fig, 2.b) [18].

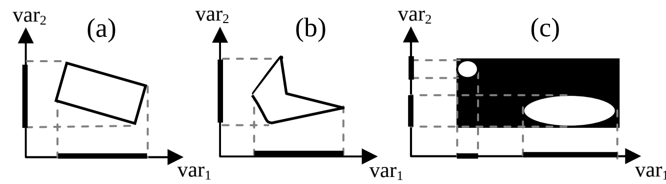


Fig. 2: Convexity of the feasible region

A worst case is presented in Fig. 2.c. where the feasible region consists of several small areas (white) scattered among the area limited by the upper and lower bounds of the decision variables, var1 and var2 (black area).

This type of feasible region, as shown later in the paper, is typical when technical constraints are imposed in the power system. The optimization algorithms in this case should have efficient exploration mechanisms so that feasible solutions can be found fast and therefore minimum computational effort is wasted wandering around in an infeasible areas.

### *C. Global Optimality*

Another aspect that tends to be overlooked in the literature is the discussion of global versus local optimality. Contrary to general opinion, this topic is not related to comparing the values of different objective functions applied to the same power system. Instead, it implies the understanding that, for a given objective function, the problem may a have a unique optimal solution, thus a local optimum is also the global optimum (Fig. 1.a) or the problem may have several local optimal points plus the global optimum. Fig. 1.b represents the latter, where the first white star represent the minimum in the interval [a, b], the second white star is the local minimum in the interval [b, c], and the black star is the global minimum in the overall interval [a, d].

This previous concept may seem superfluous, however once an optimization algorithm provides a solution, normally there are no guarantees about its quality. Proof of global optimality can be obtained but only under very specific conditions as in the case of linear programming problems [18]. In the case of MINLP problems, the capability of each algorithm to find the global optimum, without getting trapped in local minima, has to be studied separately.

## IV. Problem Description

The problem to be addressed consists of finding the optimal placement (bus number) and power rating (MVA) of multiple STATCOM units in a 45 bus system, part of the Brazilian power network (Fig. 3) [19].

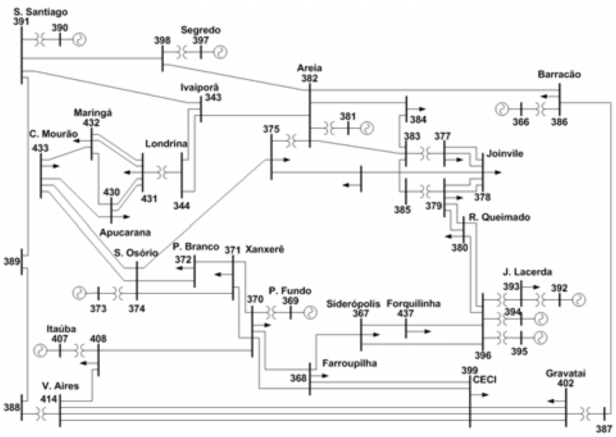


Fig. 3: Diagram of the 45 bus section of the Brazilian power system

The main objective is to minimize the bus voltage deviations throughout the power system at minimum cost. The reasons for selecting the objective criteria and specific power system are: (i) the power system is not large; therefore an exhaustive manual search can be performed to find the global optimum, (ii) the problem has a reduced, scattered and non convex feasible region, and (iii) only a steady state criterion is considered to avoid possible discrepancies if a transient analysis was also to be included [20].

### A. Objective Function

Two goals are considered: (i) to minimize voltage deviations in the system and (ii) to minimize the cost. Thus, two metrics $J_1$ and $J_2$ are defined as in (1) and (3).

$$J_1 = \sqrt{\sum_{1}^{N} (V_k - 1)^2} \quad (1)$$

where $J_1$ is the voltage deviation metric, $V_k$ is the p.u. value of the voltage at bus $k$ and $N$ is the total number of buses.

The total cost function, $C_{total}$, consists of two components: a fixed cost per unit that is installed in the system and a variable cost that is a linear function of each unit size:

$$C_{total}(M) = C_f \cdot M + C_v \cdot \sum_{p=1}^{M} \eta_p \quad (2)$$

where $M$ is the number of units to be allocated, $C_f$ is the fixed cost per unit, $C_v$ is the cost per MVA, and $\eta_p$ is the size in MVA of unit $p$.

Since $C_f >> C_v$, it is convenient to normalize each term of the cost function prior to its inclusion in the objective function:

$$J_2 = \frac{C_f \cdot M}{C_f \cdot M_{\max}} + \frac{C_v \cdot \sum_{p=1}^{M} \eta_p}{C_v \cdot M_{\max} \cdot \eta_{\max}} = \frac{M}{M_{\max}} + \frac{\sum_{p=1}^{M} \eta_p}{Max\_MVA} \quad (3)$$

where $J_2$ is the cost metric, $M_{max}$ is the maximum number of STATCOM units to be allocated, and $\eta_{max}$ is the maximum size in MVA of each STATCOM unit.

The multi-objective optimization problem can now be defined using the weighted sum of both metrics $J_1$ and $J_2$ to create the overall objective function $J$ shown in (4).

$$J = \omega_1 \cdot J_1 + \omega_2 \cdot J_2 \quad (4)$$

The weight for each metric is adjusted to reflect the relative importance of each goal. In this case, considering the maximum magnitudes of $J_1$ and $J_2$, it is decided to assign values of $\omega_1 = 1$ and $\omega_2 = 0.5$, such that both metrics have equal importance.

### B. Decision Variables

The decision variables are the location of the STATCOM units and their sizes. These variables can be arranged in a vector as:

$$x_i = [\lambda_1 \quad \eta_1 \quad \ldots \quad \lambda_M \quad \eta_M] \quad (5)$$

where $\lambda_p$, $p=1...M$, is the location (bus number) of STATCOM unit $p$. All components of the decision vector are integer numbers, thus $x_i \in Z^{2M}$.

### C. Constraints

There are several constraints in this problem regarding the characteristics of the power system and the desired voltage profile. Each constraint represents a limit in the search space, which in this particular case corresponds to:

Generator buses are omitted from the search process since they have voltage regulators to regulate the voltage.

- Bus numbers are limited to $\{1, 2, \ldots, N\}$.
- Only one unit can be connected at each bus.
- The number of units: $1 \leq M \leq 5$.
- The size of each unit: $0 \leq \eta_p \leq 250$ MVA.
- The desired voltage profile requires $N$ additional restrictions defined as:

$$0.95 \leq V_k \leq 1.05 \ , \quad \forall\, k \in \{1, 2, \ldots, N\} \quad (6)$$

Each solution that does not satisfy the above constraints is considered infeasible.

## V. Optimization Algorithms

For the optimal allocation of multiple FACTS units in a 45 bus system, three algorithms are fully developed and compared: Bender's decomposition, B&B, and Enhanced-PSO.

### A. Benders' Decomposition

This method separates two sets of decisions that are made into two consecutive stages. In the first stage of the decision making, some of the constraints are delayed to reduce the complexity of the original (master) problem. In the second stage, some of the parameters that influence the decision, whose values were originally uncertain, are known and fixed after the first decision vector is found. Thus the secondary problem is reduced in complexity and in the number of variables [21], [22].

In the case of the STATCOM allocation problem, the master problem considers the decision vector in (5) that can be naturally separated into one sub-vector for selecting optimal locations and another sub-vector for choosing optimal sizes.

The separation of the constraints can be stated as follows:

- First stage: sizes of the STATCOM units become delayed constraints, thus the reactive power limits for these devices are relaxed in the solution of the power flow. The voltage reference is set to 1.0 p.u. for each STATCOM controller. The objective function corresponds to the voltage deviation metric defined in (1).
- Second stage: with the locations of the devices determined, the set of constraints is limited to those related to the maximum size of each unit. The objective function includes the voltage deviation metric and the cost metric as in (4).

### B. Branch and Bound

B&B is a classical approach to search for an optimal solution by evaluating only a subset of the total possible solutions. The main steps in the algorithm are [18], [21], [22]:

- Branching: the set of feasible solutions is partitioned into simpler subsets. At each iteration, one of the promising subsets is chosen and an effort is made to find the best feasible solution within it.
- Bounding: the algorithm proceeds to find upper and lower bounds for the optimal objective value. There is only one upper bound u at each stage, which corresponds to the lowest among the objective values of all the feasible solutions that have appeared so far.
- Pruning: if at certain a stage, one of the subsets has a lower bound which is greater than the current upper bound, then the algorithm prunes (discards) that set.

Branching, bounding and pruning are repeated until the optimal solution is found.

For this particular problem, the objective function is defined as in (4). The branching strategy corresponds to the "depth-first search": for each subset of feasible locations, branching is performed by dividing progressively the STATCOM size intervals into smaller sub-intervals. The bounding and pruning strategies help to narrow the search by discarding as many sub-intervals as possible until the optimal value, for the particular subset of feasible locations, is found. In the next stage another subset of feasible locations is chosen, and the process is repeated until the set of all feasible locations is covered.

### C. Enhanced -PSO

#### 1) Canonical PSO formulation

The PSO algorithm considers that each particle represents a potential solution to the problem, thus the particles are defined as the decision vector in (5). The quality of the solution, that allows the best position for each particle and the swarm to be determined, is assessed using the fitness function defined in (4).

At each iteration, $t$, the position of each particle is determined by [23], [24]:

$$\vec{x}_i(t) = \vec{x}_i(t-1) + \vec{v}_i(t) \tag{7}$$

The velocity of each particle is determined by both the individual and group experiences:

$$\begin{aligned} v_i(t) = w_i \cdot v_i(t-1) + c_1 \cdot r_1 \cdot (p_i - x_i(t-1)) + ...\\ c_2 \cdot r_2 \cdot (p_g - x_i(t-1)) \end{aligned} \tag{8}$$

where $w_I$ is a positive number between 0 and 1, $c_1$ and $c_2$ are the cognitive and social acceleration constants respectively, $r_1$ and $r_2$ are random numbers with uniform distribution in the range of [0, 1]. Finally, $p_i$ is the individual best position found by the corresponding particle and $p_g$ is the global best position found by the entire swarm.

To avoid the divergence of the swarm, a maximum velocity for each dimension of the problem hyperspace is defined ($v_{max}$).

Additionally, since integer variables are included in the optimization problem, the Integer-PSO version is used, where the particle's position is rounded off to the nearest integer [24].

The PSO parameters used in this study are [1]:

TABLE I: PSO PARAMETERS

| Parameter | Optimal value |
|---|---|
| Inertia constant ($w_i$) | Linear decrease (0.9 to 0.1) |
| Individual acceleration constant ($c_1$) | 2.5 |
| Social acceleration constant ($c_2$) | 1.5 |
| $V_{max}$ for bus location | 9 |
| $V_{max}$ for STATCOM size | 50 |

#### 2) Enhanced-PSO

For this particular application, the canonical PSO algorithm described in the previous section is enhanced to facilitate the search through the problem hyperspace [1].

The additional logic in each individual is defined by the following rules:

- If the corresponding particle's best position, *pbest*, and the swarm's best position, *gbest*, are both feasible solutions then the velocity update is performed according to (8).

- If the particle has not found a feasible solution yet, then it is better to rely on the social knowledge and the velocity update equation is replaced by:

$$v_i(t) = w_i \cdot v_i(t-1) + c \cdot rand \cdot (p_g - x_i(t-1)) \quad (9)$$

where $c$ is a single acceleration constant: $c = c_1 + c_2$, and *rand* is a random number with uniform distribution in the range of [0, 1].

- If none of the particles have found a feasible solution (*gbest* and *pbest* values are both infeasible) then the velocity of each particle is updated using a random value of the maximum velocity as shown in (10).

$$v_i(t) = [r_1 \cdot v_{max}(1) \quad r_2 \cdot v_{max}(2) \quad r_3 \cdot v_{max}(3) \quad r_4 \cdot v_{max}(4)] \quad (10)$$

where $r_h$ is a random number with uniform distribution in the range of [0, 1] and $v_{max}(h)$ is the maximum velocity in the $h^{th}$ dimension of the problem hyperspace.

## VI. Simulation Results

### A. Exhaustive search

An exhaustive search is performed on the problem of optimally allocating M STATCOMS to the power system in Fig. 3 by running a power flow solution for each case in order to determine the global optimum. The solution indicates that the minimum number of devices needed to satisfy the constraints in Section IV-C is two and the computational effort corresponds to 37,196,250 power flows.

The feasible region of the problem is reduced, scattered and non convex. It is not possible to plot the entire feasible region since the dimensions are greater than three, however for illustrative purposes Fig. 4 shows the best scenario considering all possible bus locations and maximum STATCOM size of 250 MVA for each unit.

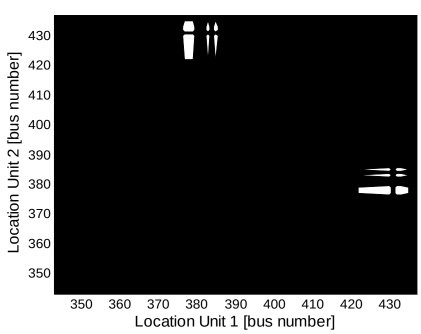


Fig. 4. Feasible region (white areas) over total problem hyperspace.

Fig. 5 shows the percentage of the feasible region with respect to the total number of cases [1].

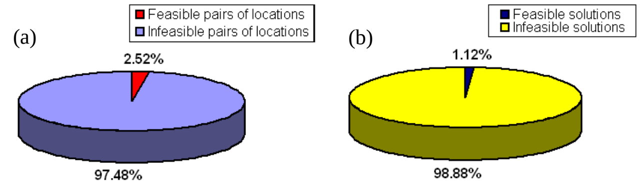


Fig. 5. (a): Percentage of feasible locations over total possible combinations. (b): Percentage of feasible solutions over total problem hyperspace.

The global optimal solution is to place one STATCOM unit of 75 MVA at bus 378 and the second unit of 92 MVA at bus 433. The effect of the two STATCOM units is shown in Fig.6.

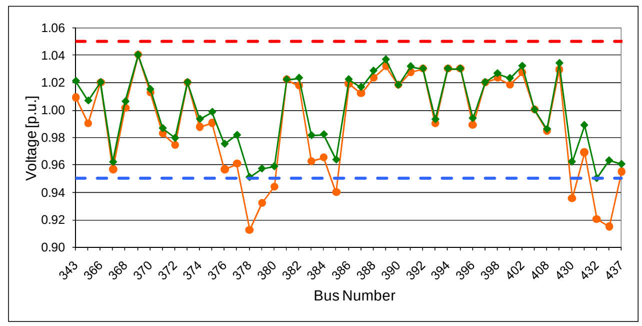


Fig. 6: Voltage profile without (-•-) and with STATCOM units (-♦-)

After the devices are optimally placed, all bus voltages are in the desired range of ±5% voltage deviation. Additionally, the voltage deviation metric J1 improves by 26.5 % from an original value of 0.2482 to 0.1824.

### B. Classical versus metaheuristic approaches

Table II summarizes the overall performance data for the classical and the Enhanced-PSO algorithms. The parameters considered for evaluating the performance of each method are the ability of the corresponding algorithm to find the global optimal solution and its computational effort.

Table II: Algorithms' performance – 45 bus system

| Parameter | Benders | B&B | Enhanced-PSO |
|---|---|---|---|
| Bus 1, Size 1 (MVA) | (378, 75) | (378, 67) | (378, 75) |
| Bus 2, Size 2 (MVA) | (433, 92) | (430, 150) | (433, 92) |
| Objective function value | 0.9174 | 1.0170 | 0.9174 |
| Voltage deviation metric | 0.1824 | 0.1819 | 0.1824 |
| Time (sec) | 18,611 | 846 | 666 |
| Power flows | 63,095 | 2,155 | 2,000 |

Considering the ability of the algorithms to find the global optimal solution, Enhanced-PSO and Bender's decomposition, are able to find the global optimum. On the other hand, the B&B algorithm gets trapped in a local minimum.

Concerning the computational effort, the number of fitness function evaluations for Benders' decomposition is 31.5 times larger than the Enhanced-PSO, with the resultant increase in computational time. Nevertheless, both algorithms require only a fraction of the total computational effort required by the exhaustive search, 0.17% and 0.005% for Benders, decomposition and Enhanced-PSO, respectively.

## VII. Conclusions

This paper compares three optimization algorithms applied to the problem of optimal allocation of FACTs devices in the power system: the classical approaches Bender's decomposition and Branch and Bound (B&B) and the metaheuristic technique Enhanced-PSO, which has been proven to be more effective than other evolutionary computation techniques to solve this type of problems [1].

Emphasis is placed on aspects of the optimization process that tend to be overlooked in the literature:

Convergence into feasible regions: for this type of application the feasible region is reduced, scattered and non-convex, therefore special consideration has to be given to the exploratory capabilities of the optimization algorithms.

Global Optimality: until now there is no proof that metaheuristic algorithms provide global optimality. This paper analyzes this aspect using a simple but realistic case study of optimal STATCOM allocation considering steady state and economic criteria. An exhaustive search is carried out on a 45 bus system to find the global optimum of the problem, so the quality of results obtained for different optimization algorithms can be evaluated.

Classical versus metaheuristic approaches: the classical approaches, Bender's decomposition and B&B are compared with the Enhanced-PSO considering the capability of the algorithms in finding the global optimal solution and their computational effort. Bender's decomposition and Enhanced PSO are capable of finding the global optimum however B&B becomes trapped in a local optimal point. For all algorithms, the number of power flow computations is small (compared to the exhaustive search), but a comparison between them favors the metaheuristic algorithm since Bender's decomposition takes 30.5% more computational effort than the Enhanced – PSO.

The previous results validate the effectiveness of the Enhanced-PSO method in solving the particular problem of optimal allocation of FACTS devices in a power system. However, a final remark is that, at present, there is no optimization method that universally outperforms all others. The selection of an algorithm is problem dependent, and this paper makes a particular effort in showing different aspects that should be considered while choosing an optimization method for any particular problem.

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