

Fuzzy Guided Constructive Heuristic Applied to Transmission System Expansion Planning

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Abstract—This work presents a constructive heuristic algorithm that uses fuzzy decision making to solve the transmission system expansion planning problem. The fuzzy system is used as a guide to circumvent some critical problems found in constructive heuristics that employ sensitivity index. The sensitivity index is derived from the resolution of relaxed models, and works as a guide to circuit addition. The heuristic presented in this paper is based on the well known branch-and-bound algorithm. Fuzzy decision making is used to decide the instant to divide the problem into two new subproblems. Tests have been conducted with part of real Brazilian systems in order to verify the efficiency of the proposed method.

Index Terms—Transmission system expansion planning, Fuzzy Decision Making, Constructive Heuristic Algorithm.

I. INTRODUCTION

THE power system operation and management have undergone through deep changes in the recent decades. The deregulation and the disaggregation of the power companies aimed the creation of a competitive market environment, in which the electric power is considered as a commodity. The planning of transmission systems can be divided in short, medium and long term plan. As the planning gets closer to a short term, detailed analysis such as those involving voltage limits, reactive power management, stability, construction details must be considered. For a long term planning, the main objective is to obtain the backbone of the power transmission. The idea is to determine the location of transmission lines and/or transformers to be installed in a specified time horizon in order to meet the desired operating conditions at minimum cost. The traditional planning, also known as centralized planning, takes into account the existence of a centralized system in a regulated monopoly structure. A simplification of the problem is obtained considering a “static” approach, which represents a planning for a single estimate of power demand. The base year topology, candidate circuits, generation and load data for the planning horizon and investment constraints, represent the basic data. A more realistic model is represented by the existence of several planning stages in which it is also necessary to determine the period when the new circuits would be installed (multistage planning). The latter formulation also represents a much more complex problem to solve.

Let us consider the centralized planning with the static approach. This case represents the less complex planning

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model. However, the mathematical modeling is still complex. It is usually formulated as a mixed integer non-linear problem. In terms of network model, usually for a long term planning the DC load flow model is used even though the ac model could be employed as well [1]. Even when the network is simplified, for instance, using the DC load flow model, it still represents a very hard problem due to the number of variables, constraints and also possibilities that increases according to the system size. As a consequence, it is regularly treated as a combinatorial problem.

Two major approaches have been found in the technical literature: 1) The problem is solved with exact methods such as the Benders decomposition and 2) by approximate methods. Methods belonging to (1) have proof of convergence and also a good performance with small and medium systems, as well as the upper and lower bounds for the solutions. However, for large systems, the convergence is obtained with more difficulty and depends on the fine tuning of the method (usually, a classical nonlinear optimization problem). Among the exact methods, the branch and bound [2] and Benders decomposition are most utilized [3], [4]. The approximate methods are represented by heuristic and metaheuristic algorithms. The principal feature is the gradual construction of the solution, which results in a less complex programming, however, the convergence to the optimal solution is uncertain. In this class of method, the constructive heuristic algorithms [5] and metaheuristics such as genetic algorithm, simulated annealing among others have been reported in the technical literature [6]–[9].

II. STATIC DC MODEL FOR LONG TERM TRANSMISSION PLANNING

When a single planning horizon is considered, the mathematical model is as follows:

$$\text{Min } v = \sum_{(i,j) \in \Omega} c_{ij} n_{ij} \quad (1)$$

s.t.

$$Sf + g = d \quad (2)$$

$$f_{ij} - \gamma_{ij}(n_{ij}^o + n_{ij})(\theta_i - \theta_j) = 0 \quad (3)$$

$$|f_{ij}| \leq (n_{ij}^o + n_{ij})\bar{f}_{ij} \quad (4)$$

$$0 \leq g \leq \bar{g}$$

$$0 \leq r \leq d \quad (5)$$

$$0 \leq n_{ij} \leq \bar{n}_{ij}$$

$$n_{ij} \text{ integer} \quad (6)$$

$$f_{ij} \text{ and } \theta_j \text{ unbounded}$$

$$(i, j) \in \Omega$$

where

v	investment cost in m.u. (monetary unit)
c_{ij}	cost of a circuit in path $i - j$ (m.u.)
θ_j	voltage angle in bus j
γ_{ij}	susceptance of the circuit in path $i - j$
n_{ij}^o	number of circuits of the original topology in path $i - j$
n_{ij}	number of circuits added into path $i - j$
\bar{n}_{ij}	maximum number of circuits in path $i - j$
f_{ij}	power flow in path $i - j$
\bar{f}_{ij}	maximum power flow limit of a circuit in path $i - j$
g_k	power generation in bus k
\bar{g}_k	maximum power generation in bus k
S	Transpose of the branch-node incidence matrix
r	vector of artificial generators (load shedding)
\bar{g}	vector of maximum generation in buses
d	vector of loads
Ω	set of all paths

Expression (1) specifies the minimization of the objective function, constraint (2) represents Kirchhoff's Current Law and constraint (3) is the Kirchhoff's Voltage Law. The rest of the constraints represent the operational limits of the components. This formulation is the same of the one presented in [3]. Another approach to deal with the difficulty in solving the earlier problem is to apply simplifications. If constraint (3) is ignored, the problem turns into the transportation model and only the active power is considered [10]. It must be noticed that the suppression of constraint (3) makes linear the mathematical model while the complete model (DC model) is nonlinear due to the product of integer and real variables in (3). We call as hybrid any intermediary model between the DC model and the transportation model. Therefore, any model that considers only part of the constraints (3), represents a hybrid model. In this context, it is possible to formulate a hybrid linear model or a hybrid nonlinear model as in [11]. In this work we have employed the hybrid linear model as will be discussed further in the next sections.

III. CONSTRUCTIVE HEURISTIC ALGORITHM (CHA)

Since Garver [10] presented the constructive heuristic algorithm with the transportation model, several new CHA have been proposed to the transmission expansion planning. Some of them are: the algorithm based on minimum effort criterion [12] and the algorithm based on minimum load shedding criterion [13]. Taking a closer look on these algorithms, one can notice that the main difference among them relies on the sensitivity index that is used to guide the search. A constructive heuristic algorithm adds one transmission line in each iteration based on specified criterion, which is usually based on a performance index. The performance index provides the information of how much the system would improve if a specific transmission line is inserted into the system. Normally, the calculation of such index requires the resolution of linear program or nonlinear program. When the stopping criterion is met, the algorithm stops. As can be observed, this is a simple strategy to determine the expansion of the system and usually it has a good performance for small systems. The critical point

of the classical CHA is that it may get stuck fastly in poor local optimal solutions.

1) *Hybrid Linear Model*: Let us consider the hybrid linear model, which has been used in this work and represents a linearized version of [11]. Differently from the DC model, in this formulation only the existing transmission lines must follow the Kirchhoff's Voltage Law. The candidate lines, which are represented by variables n_{ij} are not subject to the nonlinearity of KVL. If variable n_{ij} is considered a real variable, instead of integer variable, this model becomes a Linear Programming problem. Since lines added to the base topology must follow the two Kirchhoff's laws, this model generates solutions that are also feasible for the DC model.

$$\begin{aligned}
 \min v &= \sum_{(i,j)} c_{ij} n_{ij} & (7) \\
 \text{s.t.} & \\
 & S f + S^o f^o + g = d \\
 & f_{ij}^o - \gamma_{ij}^o (\theta_i - \theta_j) = 0 \quad \forall (i,j) \in \Omega_1 \\
 & |f_{ij}^o| \leq \bar{f}_{ij} n_{ij}^o \quad \forall (i,j) \in \Omega_1 \\
 & |f_{ij}| \leq \bar{f}_{ij} n_{ij} \quad \forall (i,j) \in \Omega \\
 & 0 \leq g \leq \bar{g} \\
 & 0 \leq n_{ij} \leq \bar{n}_{ij} \\
 & n_{ij} \text{ integer} \\
 & f_{ij} \text{ unconstrained} \\
 & \theta_j \text{ unconstrained} \quad \forall j \in \Omega_2
 \end{aligned}$$

Basically, the variables are the same of DC model, except S^o which is the branch-node incidence matrix of the existing lines of the base topology; f^o is the power flow in the circuits of the base topology; S is the branch-node incidence matrix of the candidate circuits; f is the vector of power flows through the added branches; γ_{ij}^o is the susceptance of the circuits of the base topology; θ_j are the voltage angles in buses of the base topology; Ω_1 is the set of existing circuits; Ω is the set of candidate circuits; Ω_2 is the set of all buses.

2) *Calculation of sensitivity index*: An approach to calculate the sensitivity index is to relax the integrality constraint of variables n_{ij} that represent the number of transmission lines in path $i - j$. Instead of considering it as an integer variable, it is made a real variable. According to the selected network model, the problem becomes a linear program problem (LP) or nonlinear problem (NLP). For the hybrid model shown earlier the problem becomes a LP problem. We can select the path to add a line based on the maximum value of the resulting n_{ij} variable as defined in [10]. Considering this criterion the following sensitivity index is calculated.

$$SI_{ij} = n_{ij} \bar{f}_{ij} \quad (8)$$

where n_{ij} is the value obtained from the LP. The circuit corresponding to the largest SI_{ij} is chosen, or:

$$\text{Selected path } i-j = \max\{n_{ij} \bar{f}_{ij}; n_{ij} \neq 0\} \quad (9)$$

The initial topology (or the base topology) with the addition of the new line according to the sensitivity index will form the current topology for the next iteration.

A. Deficiencies of the CHA

As pointed earlier, the CHA may find the optimal or suboptimal solutions of small or medium systems. However, for large systems the CHA based on the sensitivity index may end up far from the optimal solution. According to the reference [14], the principal reasons for the CHA to deviate the search to low quality solutions are:

- 1) Selection of high cost lines;
- 2) Selection of lines based on small values of n_{ij} .

The deviation tends to occur in two moments during the search. The first one occurs in the first iterations when high cost lines (which usually also present the highest transmission capacity) are selected. When this happens, other possibilities involving smaller lines, also cheaper ones, are rather decreased.

The second critical moment is in the last iterations, when the values of variable n_{ij} resulting from the solution of the corresponding LP or NLP problem approaches to zero. It might exist a situation in which the current topology still requires additional transmission lines, however, the decision of where to install would have been taken based on tiny values of n_{ij} . The accuracy of such choice is low because the sensitivity index gives an indication based on low values of n_{ij} and after the path $i - j$ is selected an integer value of line is added instead of the fraction of a line as indicated by the n_{ij} . In order to deal with the two situations depicted above a constructive heuristic in branch-and-bound structure with fuzzy decision making is adopted in this paper.

IV. CHA WITH FUZZY DECISION MAKING

A CHA usually converges rapidly in few iterations. However, for large systems the tendency is to converge in poor solutions. The metaheuristics usually require a considerable amount of computing time and provide good quality solutions. The same happens with classical optimization methods. As a conclusion, an algorithm that stands between the effortless programming of CHA that provides good solutions as the metaheuristics is desired. Therefore, we propose a hybrid algorithm that blends these two characteristics as in [5]. Moreover, we introduce the fuzzy decision making to adjust adaptively and dynamically the search. The main idea is to obtain high quality solutions with low computational cost.

A. Fuzzy Decision Making

The human reasoning accepts and naturally process approximate or inaccurate information gathered from different ways. However, in a computing system, the modeling and treatment of approximate information is a complex task. For example, the adjustment of the room temperature with the air conditioning system would be a simple task if we set to turn the air conditioning on when the temperature reaches 77.0°F and to turn off when it goes below 68.0°F. However, if the adjustment is made according to the sensation of the user,

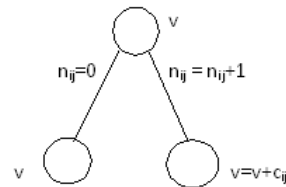


Fig. 1. Branching and creation of two subproblems

which depends on many factors, it would be totally different. For example, we would decide to turn on when the room is too “hot” and keep cooling until a “mild temperature” is reached. Fuzzy logic deals with uncertain information by treating them not as a binary information like black or white, or true or false, but also allowing the degree of truth of a condition that can range between 0 and 1. For linguistic variables which translate the state, it would exist intermediary states such as “not so cold” or “very hot”. Therefore, there is a variable grade of membership that describes such situation. Lofti Zadeh [15] proposed a theory to correctly express the grade of membership and created a translation between the world of exact representation (crisp information) and the imprecise information from the real world. We will use the concepts from Fuzzy Set Theory to improve the CHA.

The use of fuzzy sets in transmission expansion planning is increasing gradually. Most of the papers published in this topic focus on the use of fuzzy sets theory to model and to deal with the uncertainties from different operation conditions due to power market behavior, reliability. Or, to deal with some conflicting objectives that may affect the cost of the plan. Some of these papers can be found in [16], [17] In this paper, the use of fuzzy logic is slightly different and is employed to enhance the algorithm by introducing approximate logic to a conventional constructive heuristic algorithm.

1) *Motivation:* In the method proposed in [5], the algorithm has to be tuned to each system in order to provide the best performance. This task may not be easy, because it depends on the sensitivity of the user on the characteristics of the system and may result in irregular performance of the algorithm. Usually, parameter adjustment has been the trickiest part for heuristic methods and also represents the subject of criticism for those methods. The main motivation of this work was to extract the main characteristics that allow an automatic parameter adjustment of the constructive heuristic algorithm by using fuzzy logic. As a result, the method becomes less sensitive to different systems being solved, thus presenting a stable performance.

2) *Inclusion of Fuzzy Logic into the CHA:* As mentioned earlier, we will use a CHA structure similar to [5], with the aim to avoid getting trapped in local optimal solutions. The fuzzy system will make a decision to create new subproblems to test according to the value of variable n_{ij} and its relative cost. If the system considers that the selected n_{ij} for addition is very small in the problem being solved, then another subproblem (or another alternative possibility) is created (as in Figure 1) and is also tested. This strategy is based on the divide and conquer technique, which consists of dividing complex

Otherwise, solve the selected problem (LP or NLP) and calculate the objective function v_2 . Store the inferior limit of the branch sub-problem as $v_{inf} = v_1 + v_2$, and go to Step d.

d) Tree pruning tests:

After solving the sub-problem using an LP (or NLP), pruning is executed if any of the following tests are true:

Test 1: $v_{inf} \geq v^*$, where v^* is the value of the incumbent solution;

Test 2: LP (or NLP) solution is unfeasible.

Test 3: The optimal solution from the LP (or NLP) yields $v_2 = 0$. This means that a feasible solution has been found for the chosen model. In this case, verify whether the objective function of the current sub-problem (v_1) is smaller than the incumbent. If yes, then make $v^* = v_{inf}$ solution and apply Test 1 again for all candidates not pruned yet.

e) If a sub-problem was pruned in Step 4 go to Step 3, otherwise, go to Step 2.

2) Phase II

a) Simulate the removal of each added line in the base topology.

V. TESTS AND RESULTS

In order to test the efficiency of the algorithm, tests with two realistic systems have been set up: Southern Brazilian system of 46 buses, 79 circuits and demand of 6,880MW [14], and (2) reduced North-Northeastern Brazilian system of 87 buses, 183 circuits and demand of 20,600 MW (considering only stage P1) [18]. These systems represent benchmark systems and have been widely used in transmission expansion planning problems. The hybrid linear model of [11] which solutions are feasible for the DC model has been used. Despite its dimension, the Southern system presents medium complexity for this problem and the North-Northeastern system a high complexity system. The optimal solution for the Northeastern system is still unknown, even for the simplest planning model. This is due to the elevated number of isolated buses and points of high load demand.

A. Southern Brazilian System

For this system, two situations can be analyzed:

- 1) Planning with generation redispatch
- 2) Planning without generation redispatch (generation is fixed throughout the planning).

In the following the results for Southern Brazilian system with generation redispatch.

1) *Southern Brazilian System with redispatch (S1)*: An optimal solution to this system considering the DC model is \$ 70,289,000 (US\$) which was found with Benders decomposition after solving thousands of LP [3]. The CHA with fuzzy decision making found a feasible solution in 8 iterations. The CHA has found the same solution for DC model after 222 iterations. The proposed addition is the following:

$$\begin{aligned} n_{13-20} = 1, n_{20-23} = 1, n_{20-21} = 2, n_{42-43} = 1, \\ n_{46-06} = 1, n_{05-06} = 2. \end{aligned}$$

It was not possible to remove any line in Phase 2.

2) *Southern Brazilian System without redispatch (S2)*: For this case, the optimal solution is $v = 154,420,000$ (US\$), which was found by Benders Decomposition after solving thousands of LP [3].

The CHA with *fuzzy* decision making found a feasible solution after 13 iterations and the optimal solution has been found after 322 iterations with the investment value of $v = 154,420,000$ (US\$) with the following topology.

$$\begin{aligned} n_{20-21} = 1, n_{42-43} = 2, n_{46-06} = 1, n_{19-25} = 1, \\ n_{31-32} = 1, n_{28-30} = 1, n_{26-29} = 3, n_{24-25} = 2, \\ n_{29-30} = 2, n_{05-06} = 2. \end{aligned}$$

It was not possible to remove any line in Phase 2.

B. The North-Northeastern Brazilian System (NN)

The data of this system allows multistage planning (two stages - Plan P1 and P2), however, only studies considering generation without redispatch is possible. This system is known from its great complexity and the global optimal solution is still unknown [14]. One of the reasons for the complexity is the existence of many isolated buses that requires the multiple insertion of lines to make the operation feasible. The best solution published in the technical literature for the DC model is 1,360,000,000 (US\$), for plan P1, which was found after solving 300,000 LP with an enhanced version of Genetic Algorithm [19]. The CHA with *fuzzy* decision making found an expansion plan for the stage P1 with the value of 1,482,842,000 (US\$) after solving 30545 LP.

In the phase II, lines n_{69-87} and n_{27-53} have been removed. Finally the investment resulted in $v = 1,455,856,000$ (US\$), with the following topology.

$$\begin{aligned} n_{02-87} = 2, n_{03-71} = 1, n_{03-87} = 2, n_{04-05} = 1, \\ n_{04-69} = 1, n_{05-58} = 2, n_{05-68} = 1, n_{13-15} = 3, \\ n_{14-59} = 1, n_{15-16} = 2, n_{15-46} = 1, n_{16-44} = 3, \\ n_{16-61} = 1, n_{18-50} = 6, n_{18-74} = 3, n_{20-21} = 2, \\ n_{20-38} = 1, n_{22-23} = 1, n_{22-58} = 2, n_{23-24} = 1, \\ n_{25-55} = 2, n_{26-54} = 1, n_{30-31} = 1, n_{30-63} = 2, \\ n_{36-46} = 2, n_{40-45} = 1, n_{41-64} = 2, n_{43-55} = 1, \\ n_{43-58} = 1, n_{48-49} = 1, n_{49-50} = 2, n_{52-59} = 1, \\ n_{53-54} = 1, n_{54-63} = 1, n_{61-64} = 1, n_{61-85} = 2, \\ n_{67-71} = 2, n_{71-72} = 1, n_{72-73} = 1, n_{73-74} = 1. \end{aligned}$$

Considering the complexity of this system and also the number of LP solved by the algorithm, we can conclude that the method presents quality for the generation of good quality topologies that can be used to initialized advanced methods such as the metaheuristics.

The measure of a performance improvement using heuristic methods is a complicated issue because depending on the method, the strategies and the way that a candidate solution is coded (or represented) may differ completely. However, for the planning problem, we have adopted the number of LP solved as a measure of efficiency. The reason for this is due to the fact that the resolution of the mathematical model

that checks the operating conditions of a candidate solution represents the heaviest computation load of most transmission expansion algorithms in the literature. The heuristic algorithms perform several tests with different candidate solutions (in this problem, by testing several different topologies), the basic difference among all methods is how the search and the tests are conducted. The objective is always to find the best solution by using efficient logic that avoids searching the entire space of possibilities, and also avoiding get trapped in local optimal solutions. For most of the methods, despite of different strategies, the algorithm end up testing the topology to assess its quality. Therefore, it is possible to measure the efficiency of the heuristic algorithms by verifying the number of LP solved. Notice, however, that this comparison may not be applicable in cases where the computational cost for performing specific operations of a given heuristic method exceed the computational cost of the LP.

Table I summarizes the results. It shows the resulting investment (in US\$), the total number of LP solved by the algorithm, and the total execution time (Phase I and Phase II) in seconds. The program has been developed with Fortran77 code and executed in a computer with 2,00 GHz of clock and 1 Gigabyte of memory.

TABLE I

SUMMARY OF THE RESULTS, NUMBER OF LP AND WALL CLOCK TIME

System	v (10^3 US\$)	LP	Time (s)
S1	70,289	222	4
S2	154,420	461	5
NN	1,455,856	31,350	1,725

VI. CONCLUSIONS

The transmission expansion planning problem is a tough mixed integer nonlinear problem with combinatorial characteristics due to the elevated number of options and the multimodal nature. The aim of this paper was to propose a fast constructive heuristic algorithm that produces high quality solutions with less computational processing as possible. The second objective was to employ intelligent techniques to avoid frequent parameter adjustments of the algorithm. The proposed algorithm resulted very efficient. The aim was not to create an algorithm that provides the optimal solution. However, it obtained excellent results, some of them with the same quality of metaheuristics (as observed for the Brazilian Southern system) with far less computing effort. Another important feature, which have been pursuit in this work is the auto adjustment of the search parameters provided by the fuzzy decision making. This feature allows less complex analysis of any system without worrying about setting up many parameters. Different membership functions or different rules in the fuzzy system can also be employed, however, we have kept as simple as possible to keep the algorithm as a constructive heuristic algorithm. The proposed algorithm can be used as part of more sophisticate metaheuristics or it can be used as a generator of high quality solutions for any transmission planning algorithm.

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